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Three Essays in Public Finance and Environmental Economics

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Three Essays in Public Finance and Environmental Economics

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To my parents, wife, and newborn son.

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Three Essays in Public Finance and Environmental Economics

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The first essay studies the Marginal Cost of Funds in the existence of tax evasion. We develop a general equilibrium model of tax evasion, including the expected utility of taxpayers and three different revenue-raising government policies. In this rich model environment, we analytically derive the marginal cost of funds (MCF) for the alternative policy instruments. We consider two main fiscal reforms: the revision in the nonlinear tax scheme and the changes in enforcement mechanism (the audit and penalty rates). First, we derive the MCF for the tax reform and find its key determinants. The derived MCF is greater than the previous ones since it includes a "risk-bearing cost" as well as tax distortion. The reform in enforcement mechanism generates MCFs in different forms. Two more MCFs with respect to audit and penalty rates are presented. Finally, we compare these three different MCFs in numerical example and provide some policy implications.

The second essay explores optimal tax structure in the presence of status effect. When the consumption of certain goods affects one's social status, this externality creates two opposite effects in a society. Seeking higher status through "positional goods" gives individuals much incentive to suppy labor but still allocates income for less "nonpositional goods" as well. In this case, differential taxes on positional goods work as corrective instruments to internalize the social cost stemming from status seeking. Furthermore, the differential taxes generate revenue that can be used to alleviate preexisting income tax distortion. Thus, the differential taxes on positional goods could give so called "double dividend." I develop a game-theoretic model in which each individual with a different labor productivity unknown to the others engages in a status-seeking game, and the government has a revenue requirement. Then I show that, under a condition in which utility is separable between positional goods and leisure, a revenue-neutral shift in the tax mix away from nonlinear income taxes towards positional-good taxes enhances welfare. Hence, the differential taxes on positional goods are necessary together with the nonlinear income taxes for an optimal tax structure.

The third essay explores the impact of increasing capital mobility on regional growth and environment. I develop an endogenous growth model in which each local government competes against the others, to induce imperfectly mobile stock of capital into its region. Then I show that an increase in capital mobility generates "tax importing" due to which each locality experiences a higher growth rate and more degraded environment. That is, the increasing mobility dampens the capital tax and transfers the burden of pollution abatement to the locality. This finding supports the hypothesis of "race to the bottom" in environmental standards. Identifying a reduction in overall welfare of residents, I consider two alternative federal interventions in the model: uniform environmental standard and requirement of lump sum transfer or tax. Both of these federal instruments enhance the residents' welfare.

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Chapter 1

The Marginal Cost of Public Funds in the Presence of Tax Evasion

1.1 Introduction

To fund unexpected public expenses or new public projects, a government usually imposes additional distortionary taxes such as labor or capital income taxes even though the higher tax liabilities could stimulate taxpayers to cheat the government. The Internal Revenue Service (IRS) in the United States reports that the estimate of income tax liabilities not collected for 2001 is about 17%, which translates into $345 billion.[1] For cases in most other countries, the estimates are even higher. In 2003, the income tax evasion was around 25% in France and 30% in the United Kingdom.[2] Taxpayers can reduce the burden of complying with tax liability by underreporting income, whereas they have the risk of being caught in evasion which generates another welfare cost. Therefore, when a government levies distortionary taxes for public funds, the tax evasion itself matters. Taxpayers may or may not have excess burden through the behavior of tax evasion. However, the taxpayers are assumed to pay their tax liabilities fully in most economic analyses for evaluating public

[1] See Internal Revenue Service, U.S. Department of the Treasury (2006).

[2] See Christie and Holzner (2006) for more detailed discussion.

projects, even though the whole revenue from the tax liabilities is not collected in practice.

For the purpose of evaluating the public expenditures, a related literature employs a widely known concept of marginal cost of public funds (MCF), which measures the direct tax burden plus the marginal welfare cost from raising additional tax revenues.[3] However, including the prominent works of Browning (1976, 1987), almost all subsequent research does not reflect the aspects of tax evasion in MCF calculation. Even though Mayshar (1991) analytically measures the MCF for nonlinear income tax in a general equilibrium model, he does not incorporate the tax evasion as a behavioral response to a tax change. As noted by Yitzhaki (1987), the existence of tax evasion produces a risk-bearing cost in another form of excess burden. Nevertheless, he does not measure the MCF in the presence of tax evasion.[4]

Our purpose in this paper is to measure MCFs for alternative revenue-raising policies analytically when the tax evasion matters. In order to do so, we develop an analytical general equilibrium model in which taxpayers have an expected utility function, and a government imposes nonlinear income tax, audit, and fine rates, to fund public goods and lump-sum transfers.[5] Since

[3]See Browning (1976) and Mayshar (1991) for the definition of MCF. This concept originates in the argument of Pigou (1947).

[4]Slemrod and Yitzhaki (1996, 2002) mention possible ways to incorporate tax evasion into the calculation of marginal efficiency cost of funds (MECF) very broadly but do not present any analytical model, so they do not derive the MECF (or MCF) as a function of easily observable exogenous parameters.

[5]We extend a standard partial equilibrium model of tax evasion such as is presented in Allingham and Sandmo (1972).

distortionary taxes are the main instruments for public funds, it is important to know welfare cost to the use of income tax in the presence of tax evasion. Using a cost-benefit framework as presented in Mayshar (1991), we derive a "modified" MCF for nonlinear income tax ($\mathrm{MCF_T}$) in the context of tax evasion and then identify as the key determinants the expected return and variance of \$1 evaded, which consist of only audit and fine rates, implying the "riskiness of tax evasion."[6] On the other hand, a government audits more taxpayers or puts higher penalties on tax evaders to increase compliance and raise public funds.[7] Hence, we derive MCF for audit (MCF_p) and MCF for fine (MCF_θ) once more to examine welfare cost of tax enforcement policies. The remains of this work provide numerical examples of MCFs for policy recommendation for the U.S. economy.

The main contribution of this paper is to present exact MCFs that are applicable to practical use as analytic formulae. To measure $\mathrm{MCF_T}$ analytically, we endogenize the behavior of tax evasion in a model closer to the actual tax environment. This analysis is not limited to the labor-leisure choice problems as used in Browning (1976, 1987), Dreze and Stern (1990), Mayshar (1991), and Ballard and Fullerton (1992) but take actual taxpayer behaviors into account. Taxpayers mitigate the burden of tax compliance by evading tax while they bear a risk cost of being caught. The model of this paper

[6]The $\mathrm{MCF_T}$ includes tax enforcement policies as newly important parameters that the previous works with no tax evasion do not identify.

[7]Slemrod and Yitzhaki (2002) recognize that audit or penalty rates can be used to raise revenue when the tax evasion is present in a model and state that it is optimal to equalize marginal costs of raising revenue for the two alternatives at the margin.

considers not only tax distortion of labor supply but also risk-bearing cost of tax evasion, to derives $\mathrm{MCF_T}$ in a general version of those in a number of papers as listed above that do ignore tax evasion. Thus, the $\mathrm{MCF_T}$ becomes an exact measure for evaluating public expenditures. In addition, resting on the rich model environment in this paper, we obtain MCF_p and MCF_θ. The two analytic formulae together with $\mathrm{MCF_T}$ allow ones to compare the alternative revenue-raising policies on efficiency grounds. To our knowledge, this analysis is the first attempt on such a policy comparison. Therefore, the set of $\mathrm{MCF_T}$, MCF_p, and MCF_θ provides a criterion for evaluating alternative revenue-raising policies for a given level of public funds. Consequently, this paper fills the gap between the literatures on MCF and on tax evasion.

It is shown analytically that $\mathrm{MCF_T}$ is greater with tax evasion than with no tax evasion. This is due to the riskiness of tax evasion that tax enforcement policies (audit and fine rate) introduce. If the tax evasion exists, an increase in income tax rate raises both tax distortion of labor supply and riskiness of tax evasion, stimulating less labor supply but more tax evasion. When the tax evasion does not matter, an increase in income tax rate raises only the tax distortion of labor supply, however. The $\mathrm{MCF_T}$ with no tax evasion is the same as in Mayshar (1991). In this sense, this paper extends Mayshar's $\mathrm{MCF_T}$ exactly to the tax evasion case. By using the parameter values that Stuart (1984) suggests for the U.S. economy and finding proper values of audit and fine rates from the model, we show that the numerical estimate of $\mathrm{MCF_T}$

with tax evasion is 1.155 while the estimate without tax evasion is 1.076.[8] When elasticities of labor supply are positive and marginal resource cost of enforcement is sufficiently low, MCF_p and MCF_θ are less than 1, whereas MCF_T is greater than 1. If net-wage-rate elasticity of labor supply is positive, an increase in income tax rate worsens the preexisting distortions of labor supply and tax evasion.[9] On the other hand, if audit- and fine-rate elasticities of labor supply are positive, an increase in audit or fine rate alleviates the preexisting distortions. Hence, tax reform and enforcement reform could be complements rather than substitutes in this case. However, the magnitude of MCFs varies according to elasticities of labor supply and marginal resource cost of enforcement in general.

This paper is organized as follows. In the next section, we present a general equilibrium model of tax evasion that can be used to measure MCFs and find the condition under which the tax evasion exists. Sections 3 introduces a marginal revision in nonlinear income tax to derive MCF_T. In section 4, we consider a marginal change in audit and in fine to derive MCF_p and MCF_θ respectively. Using benchmark parameters that represent the U.S. economy, section 5 calculates the MCFs numerically, evaluates the alternative revenue-raising instruments, and gives policy implications. We note limitations on this analysis and offer future research in the concluding section.

[8]The numerical calculation uses the audit rate of .38 and the fine rate of 2. Furthermore, it assumes that the collected revenue is used only for a tax-neutral government project.

[9]The positive elasticity of labor supply with respect to net wage rate implies that the labor-supply curve is upward sloping.

1.2 Model

1.2.1 Taxpayer

Consider an economy in which there is a unit measure of identical taxpayers. Each individual taxpayer has a quasi-concave and twice differentiable utility function $U(C, V, G)$, where C is consumption of marketable goods, V is leisure, and G represents a publicly provided nonmarketable good. The taxpayers have three different kinds of income sources. They earn a wage at a rate w by supplying their labor L from one unit of time $(1 = L + V)$ and get an interest I by renting their stock of capital K_0 endowed. In addition to these incomes, each taxpayer receives an amount of government transfer C_0 in a lump sum fashion. However, the privately earned labor incomes above are subject to a nonlinear tax schedule T; thus, the government revenue is $R = T(wL, \phi_T)$, where ϕ_T is a vector of marginal tax rates m and an average tax rate t that the taxpayers face.[10] The taxpayers are prone to hide some of their tax liability, however, since the tax-collection agency cannot observe all the earned incomes in the economy and, thus, audits only a fraction of them. In order to prevent the taxpayers from evading their labor tax, the government employs an enforcement mechanism ϕ_E that is a pair of an auditing rate p on the population and a fine rate $\theta = 1 + \pi$ on the amount evaded, where $\pi > 0$ is the penalty rate. It is assumed that if it investigates a taxpayer's declared

[10] As in Stuart (1984) and Mayshar (1991), we assume that the government puts a tax only on the labor incomes, and therefore $T_I = 0$. This assumption allows us to compare our result to those of Stuart and Mayshar. But, it could be rather straightforward to extend to the case of nonlabor income taxation.

income, a tax-collection agency immediately discovers tax evasion. Each individual makes the decision of labor supply at the beginning of the period and, in turn, reports some portion of their labor income X to the tax-collection agency. These reports determine the after-tax income ex-ante. At the end of the period, the taxpayer's actual level of consumption becomes clear according to one of two possible states. That is, he finds his amount of consumption $C_1 = I + wL - T\,(X, \phi_T) + C_0$ when not caught evading tax as state 1, whereas $C_2 = I + wL - T(X, \phi_T) - \theta\,[R - T(X, \phi_T)] + C_0$ in the case of being caught evading tax as state 2.[11] Denoting the tax evaded $R - T(X, \phi_T)$ as E, the budget constraint at each of two states can be rewritten as

$$\begin{cases} C_1 = I + wL - R + E + C_0 & \text{if the taxpayer is not caught evading,} \\ C_2 = I + wL - R - \pi E + C_0 & \text{if the taxpayer is caught evading.} \end{cases} \tag{1.1}$$

Furthermore, when the tax collection agency audits each taxpayer at a probability of detection p, the taxpayer has an expected utility function:

$$\bar{U} = \bar{U}\,(C_1, C_2, V, G, p) = (1 - p)U(C_1, V, G) + pU(C_2, V, G). \tag{1.2}$$

1.2.2 Tax evasion and labor supply

This subsection looks into the taxpayer's decisions on labor supply and tax evasion and derives a condition under which the tax evasion exists. This condition will have an important implication on the results in the next two

[11] This setup slightly differs from Allingham-Sandmo (A-S) model. Here, the penalty paid by the taxpayer is a function of the tax evaded, whereas in the A-S model the penalty rate is on the income evaded or underreported income. See Yitzhaki (1974), Christiansen (1980), Slemrod and Yitzhaki (2002) for more detail.

sections. Regarding a market wage rate w, a nonlabor income I, and the government fiscal program Ω as exogenously given, the individual taxpayer chooses each of two possible consumption levels (C_1, C_2), a level of leisure V, and an amount of tax evasion E to maximize the expected utility in eq. (1.2) subject to two budget constraints in eq. (1.1).[12] Therefore, the corresponding Lagrangian is

$$\begin{aligned}\mathcal{L} &= (1-p)U(C_1, V, G) + pU(C_2, V, G) \\ &\quad + \lambda_1 \left[I + wL - T(wL, \phi_T) + E + C_0 - C_1\right] \\ &\quad + \lambda_2 \left[I + wL - T(wL, \phi_T) - \pi E + C_0 - C_2\right], \end{aligned} \tag{1.3}$$

where λ_1 and λ_2 are the weighted marginal utility of income according to each of two states. Setting the partial derivatives of the above Lagrangian equal to zero finds the first order conditions:

$$\mathcal{L}_{C_1} = 0 : \ (1-p)U_C(C_1, V, G) = \lambda_1, \tag{1.4}$$

$$\mathcal{L}_{C_2} = 0 : \ pU_C(C_2, V, G) = \lambda_2, \tag{1.5}$$

$$\mathcal{L}_{E} = 0 : \ \lambda_1 - \pi\lambda_2 = 0, \tag{1.6}$$

$$\mathcal{L}_{V} = 0 : \ (1-p)U_V(C_1, V, G) + pU_V(C_2, V, G) = (\lambda_1 + \lambda_2)(1-m)w. \tag{1.7}$$

[12]Before we move on to the maximization problem of the taxpayer, it is worth mentioning about the timing in the economy. First, each of the individuals faces the policy parameters (p, t, θ). Then the equilibrium labor supply, evasion and wage are determined simultaneously. Finally, government transfers and consumption level are realized. Note that policy parameters (p, t, θ) do not come from an optimization problem. In other words, government does not optimize with respect to these policies. If the government maximized total utility with respect to these policy parameters, marginal excess burden of these policies would have been the same at the optimum.

Evaluating the partial derivative $\mathcal{L}_E$ at $E = 0$ together with eqs. (1.1), (1.4), and (1.5), and then setting it greater than zero gives the condition for tax evasion to appear ($E > 0$)

$$\mu \equiv 1 - p\theta > 0, \tag{1.8}$$

where μ represents the expected payoff of one dollar evaded, $(1-p)\cdot 1 + p\cdot(-\pi)$. Note that the tax parameters ϕ_T in the nonlinear tax function do not affect whether the taxpayers evade or not. The existence of tax evasion depends only on the enforcement mechanism ϕ_E. If the expected return μ is less than or equal to zero, then the risk averse taxpayers must not evade any amount of their tax liability.[13] Just in the case of earning positive expected returns to one dollar evaded, the taxpayers evade a fraction of tax that depends on a degree of risk preference as well as a level of payoff expected. That is, the positive expected payoff in eq. (1.8) could be interpreted as a gamble favorable to the taxpayers.[14] This condition will play a key role in the two next sections. Plugging eqs. (1.4) and (1.5) into eq. (1.6) and (1.7) yields two equations

$$\frac{U_C(C_1, V, G)}{U_C(C_2, V, G)} = \frac{\pi p}{1-p}, \tag{1.9}$$

$$\frac{(1-p)U_V(C_1, V, G) + pU_V(C_2, V, G)}{(1-p)U_C(C_1, V, G) + pU_C(C_2, V, G)} = (1-m)w, \tag{1.10}$$

respectively. Eq. (1.9) shows that the marginal rate of substitution between consumptions of state 1 (not caught) and state 2 (caught) should be equal

[13] If $\mu = 1 - p\theta \leq 0$, there exists a corner solution which implies no tax evasion, $E = 0$. Thus, the condition in (1.8) guarantees an interior solution, $E > 0$.

[14] A gamble is said to be fair (unfavorable) if it has a zero (negative) expected return. See Arrow (1971) and Yitzhaki (1987) for more discussions

to the ratio of ex-ante income loss relative to gain for one dollar evaded. An increase in either probability of detection, penalty rate or both decreases tax evasion as the marginal utility of consumption in state 1 becomes relatively higher than in state 2. In eq. (1.10), the mean marginal rate of substitution between leisure and consumption is equated with net wage rate. Hence, a higher marginal tax rate or a lower wage rate acts as a disincentive to labor supply since the mean marginal utility of leisure gets relatively lower than that of consumption.

1.2.3 Firm

By borrowing a fixed stock of capital and employing a level of labor from the individuals, an aggregated firm produces market product $Y = f(K_0, L)$, where the production technology f has positive and diminishing productivity. Given the product price of one (normalized for simplicity), a wage rate, and a capital rental rate, the firm maximizes its profit $\Pi = f(K_0, L) - I - wL$, which gives the wage rate and nonlabor income as follows:

$$w = f_L(K_0, L), \tag{1.11}$$

$$I = Y - wL. \tag{1.12}$$

Then the firm's demand and the individuals' supply for labor together with the fixed level of capital determine an equilibrium wage rate and an equilibrium rental price of capital in the competitive factor markets.

1.2.4 Government

Since the size of population is measured as one, the probability of detection p implies the ratio of taxpayers caught evading tax relative to all the taxpayers. Therefore, the government revenue is

$$\begin{aligned}\bar{R} = \bar{R}(wL, E, \phi_T, \phi_E) &= (1-p)(R-E) + p(R+\pi E) - h(p,\theta) \\ &= R - (1-p\theta)E - h(p,\theta). \qquad (1.13)\end{aligned}$$

In eq. (1.13), both of labor income and tax evaded affect the revenue $\bar{R}$. Note that if all the individuals report their incomes truthfully and pay their tax liabilities, the government collects the revenue equal to $R - h(p,\theta)$. To secure a particular level of revenue, the government can employ the nonlinear income taxes ($\phi_T = (m,t)$) or force the taxpayers to pay the taxes they owe ($\phi_E = (p,\theta)$). The resource cost $h(p,\theta)$ is increasing in audit and fine rates, i.e. $h_p \geq 0$ and $h_\theta \geq 0$. The government needs some portion of the collected revenue to cover the cost of detecting tax evasion and penalizing tax evaders for dishonesty. The rest is used to finance the supply of a nonmarket good $G = g(R_G)$ and the transfer of market goods $C_0 = e(R_C)$ in which the technology of government production g and the transfer efficiency e satisfy $g' > 0 > g''$ and $e' > 0 > e''$, and two tax revenues R_G and R_C (in terms of market goods) are spent for G and C_0 respectively. Consequently, the government budget constraint becomes $R_G + R_C = \bar{R}$, and the set $\Omega = \{G, C_0, \phi_T, \phi_E\}$ stands for the government fiscal program.

1.3 Tax reform

In two consecutive sections, we consider a balanced-budget marginal revision in the government fiscal program Ω, including either a nonlinear tax reform that alters ϕ_T or an enforcement reform that alters ϕ_E and a corresponding reform in spending that changes G and C_0. Following the cost-benefit framework as in Mayshar (1991), this section first investigates the effect of a marginal revision only in the nonlinear tax schedule on the welfare of individuals. The tax reform and corresponding reform in spending $\{G, C_0, \phi_T\} \subset \Omega$ are desirable if total change in the taxpayer's expected utility is positive or equal to zero:

$$d\bar{U} = \bar{U}_{C_1} dC_1 + \bar{U}_{C_2} dC_2 + \bar{U}_V dV + \bar{U}_G dG \geq 0. \tag{1.14}$$

Even though $\bar{U}$ depends on p as shown in eq. (1.2), there is no variation in $\bar{U}$ with respect to p since a government does not consider any marginal changes in enforcement policies. Differentiate the taxpayers' two state-dependent budget constraints in eq. (1.1), the interest in eq. (1.12) and the government revenue function in eq. (1.13) to get $dC_1 = dY - dR + dE + dC_0$, $dC_2 = dY - dR - \pi dE + dC_0$, and $d\bar{R} = dR - \mu dE$. Furthermore, we have $dV + dL = 0$ from one unit of time and $dY = wdL$ by differentiating the production function and then using the wage rate in eq. (1.11). Combining these equations together with the first-order conditions in eqs. (1.4) - (1.7) gives

$$\left[\bar{U}_G / (\lambda_1 + \lambda_2)\right] dG + dC_0 \geq d\bar{R} + \mu dE - mwdL, \tag{1.15}$$

where the right- and left-hand sides of which indicate the marginal cost of tax reform and the marginal benefit of corresponding reform in spending in terms of dollar value.[15] Suppose that the government uses a share β of its marginal revenue $d\bar{R}$ to transfer the market goods and the remaining to supply the nonmarket good, i.e. $dR_C = \beta d\bar{R}$ and $dR_G = (1-\beta)d\bar{R}$. Dividing the left- and right-hand sides of eq. (1.15) by $d\bar{R} > 0$, we define the marginal benefit and cost of funds as follows:

$$\begin{aligned} \mathrm{MBF} \equiv & (1-\beta)g'(R_G)\left[\bar{U}_G/\left(\lambda_1+\lambda_2\right)\right] + \beta e'(R_C) \\ & \geq 1 + \left(\mu dE - mwdL\right)/d\bar{R} \equiv \mathrm{MCF_T}. \end{aligned} \tag{1.16}$$

The left- and right-hand sides of eq. (1.16) imply the welfare benefit and cost of the marginal tax dollar to the individuals in this economy. Note that $R = T\left(wL, \phi_T\right) = twL$. Differentiating eq. (1.13) yields the marginal government revenue:

$$d\bar{R} = (1-\gamma)twdL + wLdt - \mu dE. \tag{1.17}$$

In eq. (1.17), $\gamma = \left(dw/dL\right)\left(L/w\right) = -Lf_{LL}/f_L$ is the elasticity of wage rate with respect to labor supply. Substituting eq. (1.17) into the marginal cost of funds in eq. (1.16) leads to the following:

$$\mathrm{MCF_T} = 1 + \frac{\mu dE - mwdL}{\left(1-\gamma\right)twdL + wLdt - \mu dE}. \tag{1.18}$$

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[15] From eqs. (1.4), (1.5) and (1.7), $d\bar{U} = \lambda_1 dC_1 + \lambda_2 dC_2 + (\lambda_1+\lambda_2)(1-m)wdV + \bar{U}_G dG$. Plug $dC_1 = dY - dR + dE + dC_0$ and $dC_2 = dY - dR - \pi dE + dC_0$ into this equation to get $d\bar{U} = (\lambda_1+\lambda_2)[dY - dR + dC_0 + (1-m)wdV] + (\lambda_1 - \pi\lambda_2)dE + \bar{U}_G dG$. Since $dY = wdL$, $dV = -dL$, $d\bar{R} = dR - \mu dE$, and from eq. (1.6), $\lambda_1 - \pi\lambda_2 = 0$, we have that $d\bar{U} = (\lambda_1+\lambda_2)[dC_0 - d\bar{R} - \mu dE + mwdL] + \bar{U}_G dG$. Finally, set $d\bar{U} \geq 0$, divide this inequality by $\lambda_1 + \lambda_2$, and rearrange it to arrive at eq. (1.15).

If the expected payoff of one dollar evaded becomes less than or equal to zero ($\mu \leq 0$), each taxpayer does not evade any fraction of his tax on labor income ($E = 0$). Under this case, the taxpayer changes only his labor supply in response to the marginal revision in tax policy. Consequently, the marginal cost of funds in eq. (1.18) exactly reduces to that of Mayshar (1991). However, our analysis generalizes Mayshar's formula. We derive the marginal cost of funds, including even the circumstance where taxpayers evade their tax liabilities ($E > 0$) when the expected payoff is greater than zero ($\mu > 0$). Hence, the taxpayer changes his tax evaded as well as labor supply when a government revises a given tax rate.

We evaluate the changes in tax evasion and labor supply, i.e. dE and dL in eq. (1.18), to derive $\mathrm{MCF_T}$ in terms of exogenous parameters in this model. The evaluation of dE needs log-linearization and Taylor expansion. Taking log and differentiating eq. (1.9) totally give the following:

$$\begin{aligned}
&\left[\frac{U_{CC}(C_1,V,G)}{U_C(C_1,V,G)}dC_1 - \frac{U_{CC}(C_2,V,G)}{U_C(C_2,V,G)}dC_2\right] \\
&+ \left[\frac{U_{CV}(C_1,V,G)}{U_C(C_1,V,G)} - \frac{U_{CV}(C_2,V,G)}{U_C(C_2,V,G)}\right]dV \\
&+ \left[\frac{U_{CG}(C_1,V,G)}{U_C(C_1,V,G)} - \frac{U_{CG}(C_2,V,G)}{U_C(C_2,V,G)}\right]dG = 0.
\end{aligned}$$

In the left-side of the above equation, the second and third terms vanish out since $\frac{U_{CV}(C_1,V,G)}{U_C(C_1,V,G)} = \frac{U_{CV}(C_2,V,G)}{U_C(C_2,V,G)}$ and $\frac{U_{CG}(C_1,V,G)}{U_C(C_1,V,G)} = \frac{U_{CG}(C_2,V,G)}{U_C(C_2,V,G)}$. The equation then reduces to

$$r\left(C_1\right)dC_1 = r\left(C_2\right)dC_2, \tag{1.19}$$

where $r(\cdot) \equiv -\frac{U_{CC}(\cdot,V,G)}{U_C(\cdot,V,G)}$ denotes the absolute risk aversion or curvature of utility function.[16] Two state-dependent marginal utility functions $U_C(C_1, V, G)$ and $U_C(C_2, V, G)$ can be approximated by the first-order Taylor expansion as follows:

$$U_C(C_1, V, G) = U_C(C_2, V, G) + U_{CC}(C_2, V, G)(C_1 - C_2),$$
$$U_C(C_2, V, G) = U_C(C_1, V, G) + U_{CC}(C_1, V, G)(C_2 - C_1).$$

Dividing these two approximations by $U_C(C_2, V, G)$ and $U_C(C_1, V, G)$ respectively and using eq. (1.9) together with the fact from eq. (1.1) that $C_1 - C_2 = \theta E$, we derive

$$r(C_1) = \frac{\mu}{\pi p \theta E} \qquad \text{and} \qquad r(C_2) = \frac{\mu}{(1-p)\theta E}, \tag{1.20}$$

where the tax evaded E is assumed to be positive.[17] The absolute risk aversion at each of C_1 and C_2 is determined only by the tax evaded E as a endogenous variable and two enforcement policies p and $\theta = 1 + \pi$ as exogenous parameters.[18] Substituting the two absolute risk aversions into eq. (1.19) and using again the fact that $dC_1 = dY - dR + dE + dC_0$ and $dC_2 = dY - dR - \pi dE + dC_0$, we have

$$dE = -\left(\frac{\mu}{\sigma + \mu^2}\right)(dY - dR + dC_0), \tag{1.21}$$

[16]Take the partial derivatives of eq. (1.9) with respect to V and G, and again, divide each of the two equations by eq. (1.9) to get $\frac{U_{CV}(C_1,V,G)}{U_C(C_1,V,G)} = \frac{U_{CV}(C_2,V,G)}{U_C(C_2,V,G)}$ and $\frac{U_{CG}(C_1,V,G)}{U_C(C_1,V,G)} = \frac{U_{CG}(C_2,V,G)}{U_C(C_2,V,G)}$ respectively.

[17]Even if $E = 0$ and thus $C_1 = C_2$, the two state-dependent marginal utility functions $U_C(C_1, V, G)$ and $U_C(C_2, V, G)$ are equal to the approximations respectively, and furthermore, eq. (1.19) is satisfied.

[18]Using absolute risk aversion term itself will add two additional parameters to our calculation and will make the numerical estimation harder.

where $\sigma \equiv p(1-p)\theta^2$ represents the variance of one dollar evaded. On the right of eq. (1.21), the positive coefficient in the first parentheses measures the extent to which the taxpayers change the tax evaded when their incomes change. Since the mean and variance that the coefficient includes depend only on the audit rate p and fine rate θ, the degree of tax evasion in this economy is affected by the current state of tax enforcement. In addition, eq. (1.21) shows that the taxpayers reduce their taxes evaded when their disposable incomes get higher. Thus, the taxpayer responds to the fiscal reform in the opposite way; they evade more taxes if the government collects a revenue dR, but they evade fewer taxes if it transfers the revenue dC_0 back. Since $dY = wdL$, $dR = (1-\gamma)twdL + wLdt$, and $dC_0 = \beta e'(R_C)\, dR$, the change in tax evaded in eq. (1.21) can be rewritten as

$$\begin{aligned} dE = &- \left[\frac{(\mu/\sigma)\,(1-(1-\gamma)t\,(1-\beta e'(R_C)))}{1+(\mu^2/\sigma)\,(1-\beta e'(R_C))}\right] wdL \\ &+ \left[\frac{(\mu/\sigma)\,(1-\beta e'(R_C))}{1+(\mu^2/\sigma)\,(1-\beta e'(R_C))}\right] wLdt \end{aligned} \tag{1.22}$$

in terms of the change in labor supply and average tax rate. Since both coefficients in the first and second square brackets are positive, the change in tax evaded is negatively related to the change in labor supply but positively related to the change in tax rate. If a government increases tax rates and this causes less labor supplies, then the taxpayers could evade much more. We do not conclude at this point, however, because it has not been figured out yet whether the higher tax rate has a negative effect on labor supply.

The next step is to evaluate the change dL in labor supply. Before doing so, we adopt the virtual income concept that Hausman (1985) introduced first and Mayshar (1991) applied later. The concept is used in order to make linear budget constraints from the views of utility-optimizing individuals, even though the tax function is nonlinear.[19] In the context of tax evasion, the two state-dependent budget constraints in eq. (1.1) can be reformulated as $C_1 = (1-m)wL+E+Z$ for being caught in tax evasion and $C_2 = (1-m)wL-\pi E+Z$ for not being caught by applying a virtual income Z in the following form:

$$Z = Y - (1-m)wL - R + C_0. \tag{1.23}$$

Eq. (2.34) contains non-labor income $Y - wL$, the lump-sum transfer C_0, and the term $(m-t)wL$ generated by the nonlinear tax schedule. Since each taxpayer regards Z as exogenously given, the first order conditions in eqs. (1.4) - (1.7) and the two reformulated budget constraints give the function of labor supply as $L = L((1-m)w, p, \pi, G, Z)$. Therefore, after differentiating this function with respect to $(1-m)w$, G, Z, the change dL in labor supply becomes

$$dL = \frac{\eta L}{(1-m)w} d\left((1-m)w\right) + L_G dG + L_Z dZ, \tag{1.24}$$

where η is the uncompensated elasticity of labor supply with respect to the net wage $(1-m)w$. As in Stuart (1984) and Mayshar (1991), the publicly supplied nonmarket good G is assumed to be tax-neutral at the margin, which implies that the marginal change dG does not directly affect labor supply, $L_G \equiv 0$,

[19] See Mayshar (1991) for more detail.

or the government tax revenue.[20] Using the expenditure function approach, Appendix A.1 shows that $\eta - (1-m)wL_Z = \eta^c$ in which the superscript c indicates 'compensated,' while no superscript implies 'uncompensated.' Using this fact and the assumption that $L_G = 0$, eq. (1.24) can be rewritten as follows:

$$(1-m)(1+\gamma\eta)dL = -\eta L dm - (\eta^c - \eta)dZ/w. \tag{1.25}$$

In addition, after differentiating eq. (2.34), the marginal change in virtual income Z is given as the following:

$$\begin{aligned} dZ &= (m + \gamma(1-m))\, wdL + wLdm - (1 - \beta e'(R_C))\, dR - \beta e'(R_C)\, \mu dE \\ &= [\gamma + (1-\gamma)(m - t(1-\beta e'(R_C)))]\, wdL \\ &\quad + wL\,(dm - (1-\beta e'(R_C))\, dt) - \beta e'(R_C)\, \mu dE. \end{aligned} \tag{1.26}$$

Conducting the total differentiation until now leaves a system of three equations (1.22), (1.25) and (1.26) with three unknowns dE, dL and dZ. Substituting eq. (1.22) into eq. (1.26), and in turn, eq. (1.26) into eq. (1.25) implies the proportional change $\tilde{L}$ in eq. (1.27), in which the tilde above a variable (or parameter) represents a proportional change in the variable (or parameter).

Solving the three equations above simultaneously gives dL and dE in terms of exogenous parameters. Finally, substituting eqs. (1.22) and (1.27) into eq. (1.18), we derive the $\mathrm{MCF_T}$ for the tax reform in eq. (1.28)

$$\tilde{L} = -\frac{\left[\eta^c dm/dt - (\eta^c - \eta)\frac{(1+\mu^2/\sigma)(1-\beta e'(R_C))}{1+(\mu^2/\sigma)(1-\beta e'(R_C))}\right] dt}{(1-m)(1+\gamma\eta^c) + (\eta^c - \eta)\left[m - (1-\gamma)t\frac{(1+\mu^2/\sigma)(1-\beta e'(R_C))}{1+(\mu^2/\sigma)(1-\beta e'(R_C))} + \frac{(\mu^2/\sigma)\beta e'(R_C)}{1+(\mu^2/\sigma)(1-\beta e'(R_C))}\right]}, \tag{1.27}$$

[20] This assumption is ensured when the private goods is weakly separable from the public good G in the expected utility function $\bar{U}$. For example, if the conventional utility function $U(C, V, G)$ is additive or multiplicative, then the expected utility function $\bar{U}(C_1, C_2, V, G, p) = (1-p)U(C_1, V, G) + pU(C_2, V, G)$ can satisfy the property.

$$\mathrm{MCF_T} = 1 + \frac{\left(\mu^2/\sigma + m\right)\eta^c dm/dt + \left[\left(\mu^2/\sigma\right)\left(a + b\eta^c dm/dt\right) - \left(\mu^2/\sigma + m\right)\left(\eta^c - \eta\right)\right]\left(1 - \beta e'(R_C)\right)}{a + b\eta^c dm/dt + \left(\mu^2/\sigma + m\right)\left(\left(\eta^c - \eta\right) - \eta^c dm/dt\right)}, \tag{1.28}$$

where $a \equiv (1 - m)(1 + \gamma\eta^c)$ and $b \equiv m - (1 - \gamma)\,t$.

The $\mathrm{MCF_T}$ is only measured in terms of exogenous parameters. As seen in eq. (1.18), $\mathrm{MCF_T}$ in eq. (1.28) also reduces to Mayshar's for $\mu^2/\sigma = 0$ (i.e., no tax evasion). It is possible to show that the $\mathrm{MCF_T}$ in eq. (1.28) is greater than Mayshar's (See the equation 17 in his paper) when $dm/dt > 0$.[21] This is logical because the existence of tax evasion makes the economic environment uncertain and, in turn, causes an additional burden to the economy. Our $\mathrm{MCF_T}$ includes both the labor supply distortion of tax and the risk bearing cost of tax evasion. In eq. (1.28), we see that $\mathrm{MCF_T}$ has a direct cost of one dollar plus an additional term representing the labor supply distortion and risk bearing cost together. Yitzhaki (1987) points out that the total excess burden of risk and tax distortion can be treated separately if utility function is separable in consumption and labor. We do not restrict the utility function to a specific form in our model. Therefore, risk bearing cost and tax distortion are interrelated in our model.

[21]Note that $dm/dt > 0$ is a sufficient but not a necessary condition to show analytically that our $\mathrm{MCF_T}$ is greater than Mayshar's. Stuart (1984) assumes that ratio of marginal tax rate and average tax rate is constant and greater than 1. In progressive tax system for a given income level average tax rate is always lower than the marginal tax rate. However, whether dm is greater than dt or not depends on how the tax system is changed.

1.4 Enforcement reform

In the previous section, the government revises the nonlinear tax schedule for extra revenue to finance its additional expenditure. Even though the tax scheme is used as a common policy instrument, the government could employ the audit rate or the fine rate instead. For example, the IRS increased the audit rate on individuals with more than $100,000 of income by 40% in 2004.[22] This kind of practice often attracts a new analysis on measuring the costs of enforcement policies. In addition, the analysis can allow one to compare the costs of enforcement and tax policies together. Hence, this section derives two different MCFs when a tax collection agency could use either the audit rate p or the fine rate θ in the enforcement mechanism ϕ_E. Although the two MCFs are derived separately at the end, we first consider the revision in both the audit and fine rate by some point in this section and then assume only the change in each of the enforcement policies from that point on. Following the same definitions and steps used in the section above, we continue to exploit total differentiation in order to investigate the effects of tax enforcement changes on individuals' welfare. Now, the enforcement reform and corresponding reform in spending $\{G, C_0, \phi_E\} \subset \Omega$ are desirable if total change in the taxpayer's expected utility is positive or equal to zero:

$$d\bar{U} = \bar{U}_{C_1} dC_1 + \bar{U}_{C_2} dC_2 + \bar{U}_V dV + \bar{U}_G dG + \bar{U}_p dp \geq 0. \tag{1.29}$$

[22]Give reference at http://www.nytimes.com/2004/11/19/business/19irs.html.

Eq. (1.29) includes $\bar{U}_p dp$ as one more term than eq. (1.14). In contrast to the tax reform, the revision in audit rate could directly affect the taxpayer's utility function since the uncertainty of this economy has been generated by the government's random audits. Differentiate two state-dependent budget constraints of individuals in eq. (1.1), the market interest in eq. (1.12) and the expected revenue of government in eq. (1.13) to have $dC_1 = dY - dR + dE + dC_0$, $dC_2 = dY - dR - \pi dE - Ed\pi + dC_0$, and $d\bar{R} = dR - \mu dE + (\theta E - h_p)\, dp + (pE - h_\theta)\, d\theta$. We know from the previous section that $dV = -dL$ and $dY = wdL$. Combining these indentities together with the first-order conditions in eq. (1.4) - (1.7) yields the following:[23]

$$\begin{aligned} \frac{\bar{U}_G}{\lambda_1 + \lambda_2} dG + dC_0 \geq & d\bar{R} + \mu dE - mwdL + \left(h_p - \theta E - \frac{\bar{U}_p}{\lambda_1 + \lambda_2} \right) dp \\ & + \left(h_\theta - pE + \frac{\lambda_2 E}{\lambda_1 + \lambda_2} \right) d\theta. \end{aligned} \tag{1.30}$$

Eq. (1.30) implies that the marginal cost of enforcement reform on the right should be less than or equal to the marginal benefit of corresponding reform in spending on the left. The term $\bar{U}_p = -U\left(C_1, V, G\right) + U\left(C_2, V, G\right)$ can be approximated to $-U_C\left(C_2, V, G\right)\left(C_1 - C_2\right)$ by the first-order Taylor expansion. Using this approximation together with eqs. (1.1), (1.5) and (1.6) and dividing

[23]From eqs. (1.4), (1.5), and (1.7), $d\bar{U} = \lambda_1 dC_1 + \lambda_2 dC_2 + (\lambda_1 + \lambda_2)(1 - m)wdV + \bar{U}_G dG + \bar{U}_p dp$. Substitute $dC_1 = dY - dR + dE + dC_0$ and $dC_2 = dY - dR - \pi dE - Ed\pi + dC_0$ into this equation to get $d\bar{U} = (\lambda_1 + \lambda_2)(dY - dR + dC_0 + (1 - m)wdV) + (\lambda_1 - \pi\lambda_2)dE - \lambda_2 Ed\pi + \bar{U}_G dG + \bar{U}_p dp$. Since $dY = wdL$, $dV = -dL$, $d\bar{R} = dR - \mu dE + (\theta E - h'(p))dp + (pE - i'(\theta))d\theta$, $d\pi = d\theta$, and from eq. (1.6), $\lambda_1 - \pi\lambda_2 = 0$, we have that $d\bar{U} = (\lambda_1 + \lambda_2)[dC_0 - d\bar{R} - \mu dE + mwdL + (\theta E - h'(p))dp + (pE - i'(\theta))d\theta] - \lambda_2 Ed\theta + \bar{U}_G dG + \bar{U}_p dp$. Finally, set $d\bar{U} \geq 0$, divide this inequality by $\lambda_1 + \lambda_2$, and rearrange it to arrive at eq. (1.30).

each side of eq. (1.30) by $d\bar{R} > 0$, we define the marginal benefit and cost of funds as

$$\begin{aligned} \text{MBF} \equiv & (1-\beta) g'(R_G) \frac{\bar{U}_G}{\lambda_1 + \lambda_2} + \beta e'(R_C) \\ & \geq 1 + \frac{\mu dE - mwdL + (h_p - \theta E + E/p)\, dp + (h_\theta - pE + E/\theta)\, d\theta}{d\bar{R}} \\ & \equiv \text{MCF}_{\text{E}}, \end{aligned} \tag{1.31}$$

where the government is assumed again to spend the fraction β of its marginal expected revenue on the transfer. Compared to eq. (1.16), the marginal cost of funds is quite different, while the marginal benefit of funds is still same in eq. (1.31). Since there is no change in tax rates, $dR = (1-\gamma)twdL$. Thus, the marginal change in the government's expected revenue in eq. (1.13) becomes

$$d\bar{R} = (1-\gamma)twdL - \mu dE + (\theta E - h_p)\, dp + (pE - h_\theta)\, d\theta. \tag{1.32}$$

After inserting eq. (1.32) into the right-hand side of eq. (1.31), the marginal cost of funds associated with the change in probability of detection p and fine rate θ becomes

$$\text{MCF}_{\text{E}} = 1 + \frac{\mu dE - mwdL + E\,(ph_p/E - p\theta + 1)\,\tilde{p} + E\,(\theta h_\theta/E - p\theta + 1)\,\tilde{\theta}}{(1-\gamma)twdL - \mu dE + E\,(p\theta - ph_p/E)\,\tilde{p} + E\,(p\theta - \theta h_\theta/E)\,\tilde{\theta}}. \tag{1.33}$$

Through the same steps used in Section 4, we will evaluate the changes dL and dE in response to the reform. As the first step, the change in tax evasion dE is derived in terms of the change in labor supply dL. After taking the log and totally differentiating eq. (1.9), use the fact that $\frac{U_{CV}(C_1,V,G)}{U_C(C_1,V,G)} =$

$\frac{U_{CV}(C_2,V,G)}{U_C(C_2,V,G)}$ and $\frac{U_{CG}(C_1,V,G)}{U_C(C_1,V,G)} = \frac{U_{CG}(C_2,V,G)}{U_C(C_2,V,G)}$ to arrive at

$$r\,(C_1)\,dC_1 + \left(\frac{p}{1-p}\right)\tilde{p} = r\,(C_2)\,dC_2 - \tilde{p} - \left(\frac{\theta}{\pi}\right)\tilde{\theta}.$$

Plugging eq. (1.20) into eq. (1.19) with the fact that $dC_1 = dY - dR + dE + dC_0$ and $dC_2 = dY - dR - \pi dE - Ed\pi + dC_0$ gives

$$dE = -\left(\frac{\mu}{\sigma+\mu^2}\right)\left[(dY - dR + dC_0) + E\left(\frac{\sigma}{\mu^2}\right)\left(\left(1-\frac{p\theta\mu}{\sigma}\right)\tilde{p} + \tilde{\theta}\right)\right]. \tag{1.34}$$

In eq. (1.34), the second term in square brackets shows that the changes in p and θ decrease the amount of tax evaded. This implies that both the audit and fine rate determine the existence as well as the degree of tax evasion. The enforcement policies directly affect the evasion, whereas the tax codes indirectly affect the evasion through the income change as in eq. (1.21).

From this point, we consider only the change in each separate enforcement policy. The government is assumed to revise either of two enforcement policies p and θ. Let us introduce a function that makes both of the analyses simple. For $k \in \phi_E$, the function ξ_k is defined by

$$\begin{cases} \xi_p = p\theta\mu/\sigma & \text{if } dp \neq 0 \text{ and } d\theta = 0, \\ \xi_\theta = 0 & \text{if } d\theta = 0 \text{ and } d\theta \neq 0. \end{cases}$$

Then two separate analyses can be united because they have perfect symmetry except for the term that ξ_k implies. The second term in square brackets in eq. (1.34) points out the only difference between two separate reforms. A revision in audit rate directly reformulates individuals' preference orderings, since the audit rate as a policy parameter generates the uncertainty in this economy. Therefore, based on the fact that $dY = wdL$, $dR = (1-\gamma)twdL$, and $dC_0 = \beta e'(R_C)\, dR$, the change dE for each of the two reforms becomes

$$\begin{aligned} dE = &- \left[\frac{(\mu/\sigma)\left(1-(1-\gamma)t\left(1-\beta e'(R_C)\right)\right)}{1+(\mu^2/\sigma)\left(1-\beta e'(R_C)\right)}\right] wdL \\ &- E\left[\frac{\left(1-\xi_k\right)/\mu + (\mu/\sigma)\left(p\theta - kh_k/E\right)\beta e'(R_C)}{1+(\mu^2/\sigma)\left(1-\beta e'(R_C)\right)}\right]\tilde{k}. \end{aligned} \tag{1.35}$$

Next, we evaluate the change in labor supply dL by replicating the steps employed in the previous section. Totally differentiating the labor supply function $L = L((1-m)w, p, \pi = \theta - 1, G, Z)$ yields

$$dL = \frac{\eta L}{(1-m)\,w} d\left((1-m)\,w\right) + \varepsilon_k L\tilde{k} + L_G dG + L_Z dZ, \tag{1.36}$$

where ε_k is the uncompensated elasticity of labor supply with respect to the enforcement policy $k \in \phi_E$. Again, using the assumption that G is tax-neutral at the margin and totally differentiating the virtual income in eq. (2.34) respectively, the total effects dL and dZ become the following:

$$(1-m)\left(1+\gamma\eta\right)dL = (1-m)\,L\varepsilon_k\tilde{k} - \left(\eta^c - \eta\right)dZ/w, \tag{1.37}$$

$$\begin{aligned} dZ &= \left(m + \gamma\left(1-m\right)\right)wdL - \left(1-\beta e'(R_C)\right)dR \\ &\quad - \beta e'(R_C)\left(\mu dE - E\left(p\theta - kh_k/E\right)\tilde{k}\right) \\ &= \left[\gamma + (1-\gamma)\left(m - t\left(1-\beta e'(R_C)\right)\right)\right]wdL \\ &\quad - \beta e'(R_C)\left(\mu dE - E\left(p\theta - kh_k/E\right)\tilde{k}\right). \end{aligned} \tag{1.38}$$

Finally, as in the previous sections, three equations (1.35), (1.37) and (1.38) yield the change in labor supply and the $\mathrm{MCF_E}$ as

$$\tilde{L} = \frac{\left[(1-m)\,\varepsilon_k - (\eta^c-\eta)\varphi\left(1-\xi_k + (1+\mu^2/\sigma)\left(p\theta - kh_k/E\right)\right)\frac{\beta e'(R_C)}{1+(\mu^2/\sigma)(1-\beta e'(R_C))}\right]\tilde{k}}{(1-m)\left(1+\gamma\eta^c\right) + \left(\eta^c-\eta\right)\left[m - (1-\gamma)t\frac{(1+\mu^2/\sigma)(1-\beta e'(R_C))}{1+(\mu^2/\sigma)(1-\beta e'(R_C))} + \frac{(\mu^2/\sigma)\beta e'(R_C)}{1+(\mu^2/\sigma)(1-\beta e'(R_C))}\right]}, \tag{1.39}$$

$$\mathrm{MCF}_{\mathrm{E(k)}} = 1 - \frac{\left\{ \begin{array}{c} (1-m)\left(\mu^2/\sigma + m\right)\varepsilon_k^c + \varphi c_k a \\ + \left[\left(\mu^2/\sigma\right)\left((1-m)\, b\varepsilon_k^c - \varphi a\right) + \left(\eta^c - \eta\right)\varphi\left(\left(\mu^2/\sigma + m\right) + c_k b\right)\right] \left(1 - \beta e'\left(R_C\right)\right) \end{array} \right\}}{(1-m)\left(\mu^2/\sigma + (1-\gamma)\, t\right)\varepsilon_k + \varphi\left(1 + c_k\right)\left(a + \left(\eta^c - \eta\right) b\right)}, \tag{1.40}$$

where $\varphi \equiv E/wL = -(1-m)\left(\varepsilon_k^c - \varepsilon_k\right) / \left(\eta^c - \eta\right)$ for $k \in \phi_E$ and $c_k \equiv \left(1 + \mu^2/\sigma\right)\left(p\theta - kh_k/E\right) - \xi_k$.

In a world with tax evasion and positive tax rates there are two different sources of deadweight loss (or inefficiency). One is distorted labor supply because of labor tax. The other is the risk cost of evading tax. If we assume that wage, audit and fine elasticity of labor supply are greater than zero (η, ε_p, and $\varepsilon_\theta > 0$), then the following results hold. Increasing tax rates (tax reform) causes a decrease in labor supply and an increase in tax evasion ($dL < 0$ and $dE > 0$) as can be seen in the above equations. Hence, tax reform worsens preexisting tax distortions. $\mathrm{MCF}_{\mathrm{T}}$ has a direct resource cost of one dollar as well as additional deadweight loss. Thus, $\mathrm{MCF}_{\mathrm{T}}$ is greater than 1. However, enforcement reform (i.e., increasing audit or fine rate) causes an increase in labor supply and a decrease in tax evasion ($dL > 0$ and $dE < 0$). This means preexisting distortions caused by tax are lowered by enforcement reform. $\mathrm{MCF}_{\mathrm{E}}$ has a direct cost of one dollar, marginal resource cost of enforcement, and negative deadweight loss. For this reason, when the enforcement reform is costless or has a sufficient low cost, $\mathrm{MCF}_{\mathrm{E}}$ can be less than one. Enforcement could actually be a very useful policy, if costs associated with increasing audit or fine rates are low. Increasing enforcement will lower preexisting distortions and yield more extra revenue. The government can then use the revenue to lower other distortionary taxes. This argument is very similar to the double-dividend hypothesis of Bovenberg and de Mooij (1994).[24] The first dividend is decreased labor supply and tax evasion distortions while the second dividend is obtained by using tax revenue to lower some distortionary taxes.

If extra revenue from either enforcement or tax reform is used in the same way, and the enforcement has a low marginal resource cost, $\mathrm{MCF_T}$ is greater than one while $\mathrm{MCF_E}$ is less than one. In this case, the enforcement reform is superior to tax reform. In that sense, tax reform and enforcement reform are complements rather than substitutes in terms of efficiency. In other words, a tax reform increases the excess burden while an enforcement reform decreases excess burden in the economy. Thus, a tax reform should be accompanied by an enforcement reform to minimize the extra burden caused by tax reform. A very high marginal cost for enforcement or negative enforcement elasticity of labor supply may indeed cause tax and enforcement reform to be substitutes.

Government can increase enforcement in two ways. It can increase the audit or fine rates. It is not easy to see which enforcement policy has a lower MCF from the above equations. However, the terms c_p, c_θ, ε_p, and ε_θ play an important role in determining the magnitude of MCFs. Therefore, by comparing these terms, we can say which policy has lower MCF. We mentioned above

[24]Double-dividend is the notion that environmental taxes can both reduce pollution (the first dividend) and reduce the overall economic costs associated with the tax system by using the revenue generated to displace other more distortionary taxes that slow economic growth at the same time (the second dividend).

that probability and penalty elasticities of labor supply mainly determine the response of labor supply, dL, and it is positive if ε_p and $\varepsilon_\theta > 0$. The bigger the elasticity, the greater the labor supply response. Therefore, a higher elasticity means a higher decrease in tax distortion and a lower MCF. The marginal cost of increasing the audit and fine rates is included in the terms c_p and c_θ. As marginal cost of enforcement (audit and fine) increases, both MCF_p and MCF_θ increase. Everything the same, the enforcement policy that has higher elasticity of labor supply and lower marginal cost will have a smaller MCF. We can also say that the audit and fine rates are substitutes for each other in terms of efficiency of policy. They both decrease preexisting labor supply distortion (conditional on ε_p and $\varepsilon_\theta > 0$) and yield extra government revenue.

1.5 Numerical analysis

In this section, we calculate MCFs for alternative policies numerically. To estimate MCF_T, we need values for 8 parameters: η^c, η, t, m, dm/dt, γ, β, p, θ. In Table 1.1, Stuart (1984) suggests the following benchmark parameters for U.S. economy: $\eta = 0$, $\eta^c = \eta - .2$, $\gamma = .28$, $m = .427$, $m/t = dm/dt = 1.564$. In addition, we assume for now that $\beta = 0$. To find values for audit rate p and fine rate θ, we first solve the model for a specific utility function. Then, to find proper values for audit and fine rates, we match the tax evasion ratio that our model estimates with the real world tax evasion ratio, 17%.[25] We

[25]IRS estimate of tax evasion for 2001 is 17%. U.S department of Treasury, Internal Revenue Service (2006)

assume separable CRRA utility function

$$U(C, V) = \frac{C^{1-\delta}}{1-\delta} + \frac{V^{1-\epsilon}}{1-\epsilon}, \tag{1.41}$$

where V is leisure and we normalize time endowment to 1 so that $V = (1-L)$. We assume utility function parameters δ and ϵ are equal to 2. We solve our model with specified utility function and given parameter values. Equilibrium values of labor supply and tax evasion imply that when $p = .38$, $\theta = 2$, the tax evasion ratio is 17%.[26] To estimate MCF_E, we need values for ε_p, ε_θ, h_p, h_θ, and E more. In other words, we need labor supply elasticity with respect to audit and fine rate, marginal resource cost of audit and fine rate, and the amount of evasion. We find E, ε_p and ε_θ from the solution of our model with utility function above.[27]

In Table 1.2, we compare our MCF_T estimates with those of Mayshar (1991) for different government policy and individual parameter values. We analytically showed above that our MCF_T estimate is greater than Mayshar's. The estimates in the first column are based on our MCF_T formula. The second column estimates are based on Mayshar's formula. In the first row of Table 1.2, we present MCF_T estimation for different values of β. When

[26]There many values for p and θ that gives 17% evasion rate. We fix penalty rate to 1 and get $p = .38$. IRS penalty rate for tax evasion varies between 25% and 75% depending on the nature of the tax underpayment, so fixing penalty rate to 1 makes sense.However, auditing rate in the US is much lower than 38%. In the real world auditing rate is not exogenous to taxable income. Also it is much harder for wage earners to evade compared to self employed. Since our model does not consider this aspect of tax evasion, 38% auditing rate seems reasonable.

[27]Tax evasion is .04, labor supply is .56 and taxable income is .23 in equilibrium in our model. Note that we normalize time endowment and wage to 1.

Table 1.1: Benchmark parameters for the U.S. economy

Stuart (1984)	:	$\eta = 0$
		$\eta^c = \eta - .2$
		$\gamma = .28$
		$m = .427$
		$m/t = dm/dt = 1.564$
Baseline	:	$\varepsilon_p = \varepsilon_\theta = 0$
		$\beta = 0$
		$h_p = h_\theta = 0$
Model	:	$\theta = 2$
		$p = .38$
		$E = .04$

$\beta = 0$, no government revenue is transferred to taxpayers, and when $\beta = 1$, all tax revenue is transferred to taxpayers. We see that our $\mathrm{MCF_T}$ estimate is greater than Mayshar's no matter how the extra tax revenue is spent by the government. In the second row, we change the marginal tax rate, while in the third and fourth row we change audit and fine rates respectively. It is not surprising that a higher marginal income tax rate leads to higher $\mathrm{MCF_T}$ for both our estimate and Mayshar's. However, only labor supply is distorted in Mayshar's case, while both labor supply and tax evasion are distorted in our case. More enforcement (higher audit and fine rates) means less $\mathrm{MCF_T}$ because more enforcement causes less evasion in equilibrium. In the third row, for $p = .45$, our $\mathrm{MCF_T}$ estimate is very close to Mayshar's since people have almost no incentive to evade when $p = .45$ (expected return on evading, $1 - p\theta$, is still positive but very close to 0). We change labor supply elasticity in the fifth row. As elasticity increases, labor supply distortion of a tax increase becomes more severe, and this leads to higher $\mathrm{MCF_T}$ for both our and Mayshar's model.

Table 1.2: $\mathrm{MCF_T}$ with tax evasion versus without tax evasion

	$\mathrm{MCF_T}$	
	Tax evasion	No tax evasion
Benchmark case	1.155	1.076
Government policies		
1. Share of marginal revenue		
$\beta = .618$	1.210	1.160
$\beta = 1$	1.244	1.211
2. Marginal tax rate		
$m = .350$	1.131	1.055
$m = .460$	1.167	1.087
3. Audit rate		
$p = .2$	1.870	1.076
$p = .45$	1.070	1.076
4. Fine rate		
$\theta = 2.3$	1.092	1.076
$\theta = 2.6$	1.078	1.076
Taxpayer		
5. Net-wage-rate elasticity		
$\eta = .318$	1.641	1.447
$\eta = .5$	1.986	1.687

Table 1.3 compares the MCFs for three alternative revenue-raising policies (tax, audit, and fine rates), depending on different government policy and taxpayer parameters. In general, $\mathrm{MCF_T}$ is greater than MCF_p and MCF_θ in Table 1.3. Audit and fine rates (p, θ) determine the riskiness of tax evasion (μ^2/σ). More riskiness means that the risk cost of evading tax is greater. Hence, as seen in the third and fourth rows, either an increase in p and θ increases MCFs, raising the riskiness of tax evasion. A higher marginal resource cost of tax enforcement (h_p, h_θ) means higher MCF_p and MCF_θ. In fifth row,

when $h_p = h_\theta = 5$, MCF_p and MCF_θ are 1.754 and 1.423 respectively, while $\mathrm{MCF}_{\mathrm{T}}$ is 1.155. Therefore, if increasing enforcement marginally is costly, then tax reform has a lower marginal cost compared to enforcement reform. Elasticities of labor supply also play a key role in determining the magnitude of MCFs for different policies. In the 7th row, when $\varepsilon_p = \varepsilon_\theta = -.5$, MCF_p and MCF_θ are greater than $\mathrm{MCF}_{\mathrm{T}}$. MCF_p and MCF_θ are 1.397 and 1.189 respectively, while $\mathrm{MCF}_{\mathrm{T}}$ is 1.155. Elasticities have an opposite effect on $\mathrm{MCF}_{\mathrm{T}}$ and $\mathrm{MCF}_{\mathrm{E}}$. While higher wage elasticity (η) causes more labor supply distortion in the tax reform case, higher enforcement elasticities (ε_p, ε_θ) mean less tax distortion for labor supply in the enforcement reform case.

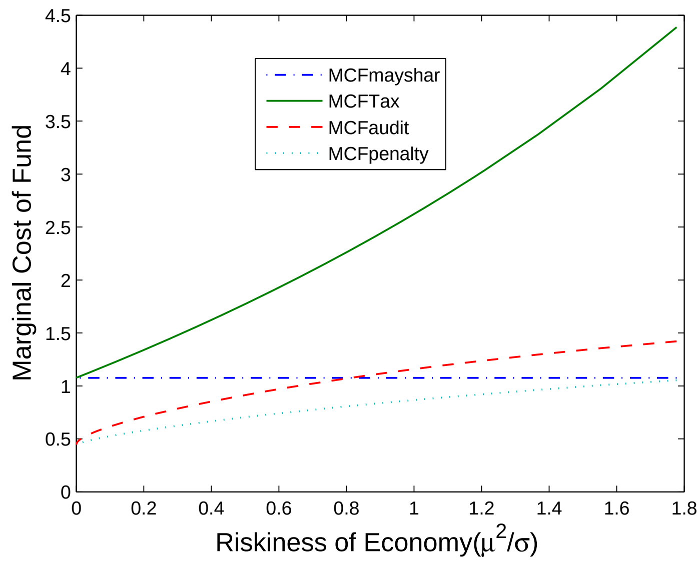
In Figure 1.1, we graph MCFs for different values of μ^2/σ when $\beta = 0$, $h_p = h_\theta = 0$. In our formulation of MCFs above, the term μ^2/σ represents the riskiness of tax evasion. Note that μ is the expected return on evading one dollar and σ is variance of return. As the audit rate or fine rate goes down, μ increases while σ decreases, and thus, overall μ^2/σ increases. In other words, when auditing becomes less common or when fines on evasion are lower, expected return on tax evasion will be greater, and taxpayers will evade more in equilibrium. Thus, the taxpayers' response dE to a policy reform will be higher (since expected return is higher), leading to more distortion. $\mathrm{MCF}_{\mathrm{T}}$ is greater than $\mathrm{MCF}_{\mathrm{E}}$ for all values of μ^2/σ, since the enforcement is assumed to be costless. Figure 1.2 shows how $\mathrm{MCF}_{\mathrm{E}}$ changes as marginal resource cost of enforcement (h_p, h_θ) increases. When the enforcement policy to deter evasion becomes more costly, $\mathrm{MCF}_{\mathrm{E}}$ increases tremendously. This is trivial because our

Table 1.3: MCFs with tax evasion for alternative revenue-raising policies

	MCF_T	MCF_p	MCF_θ
Benchmark case	1.155	.657	.587
Government Policies			
1. Share of marginal revenue			
$\beta = .618$	1.210	.704	.633
$\beta = 1$	1.245	.733	.662
2. Marginal tax rate			
$m = .35$	1.131	.657	.587
$m = .46$	1.167	.657	.587
$m = .35$ $(\beta = 1)$	1.183	.703	.633
$m = .46$ $(\beta = 1)$	1.276	.748	.676
3. Audit rate			
$p = .2$	1.870	1.250	.961
$p = .45$	1.070	.555	.529
4. Fine rate			
$\theta = 2.5$	1.065	.521	.513
$\theta = 3$	1.033	.453	.469
5. Marginal resource costs of audit and fine rate			
$h_p = h_\theta = .5$	1.155	.674	.624
$h_p = h_\theta = 2$	1.155	.877	.767
$h_p = h_\theta = 5$	1.155	1.754	1.423
Taxpayer			
6. Net-wage-rate elasticity			
$\eta = .318$	1.641	.657	.587
$\eta = .5$	1.986	.657	.587
$\eta = .318$ $(\beta = 1)$	1.725	.725	.654
$\eta = .5$ $(\beta = 1)$	2.060	.720	.650
7. Audit- and fine-rate elasticities			
$\varepsilon_p = \varepsilon_\theta = -.5$	1.155	1.397	1.189
$\varepsilon_p = \varepsilon_\theta = .5$	1.155	.433	.366

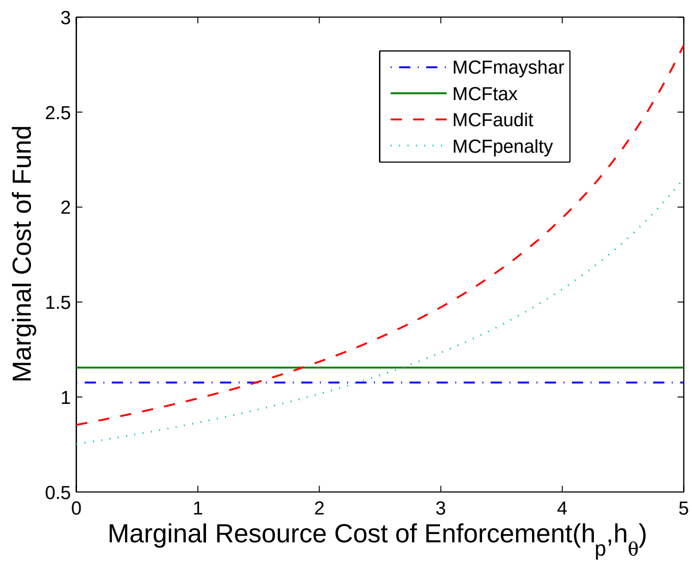
MCF_E includes the resource cost of increasing tax enforcement as well as labor supply distortion and risk-bearing cost. Thus, when policymakers decide which policy to use to raise additional tax revenue, they need to analyze carefully

Figure 1.1: MCFs and riskiness of tax evasion



how much increasing tax enforcement costs. In Figure 1.3 , we see how MCFs change with public spending policy. As β increases, all MCFs go up. When $\beta = 1$, extra tax revenue is returned to individuals. This compensates the income effect of the tax or enforcement reform. As a result, when β goes up, labor supply distortion increases, causing MCFs to go up as well.

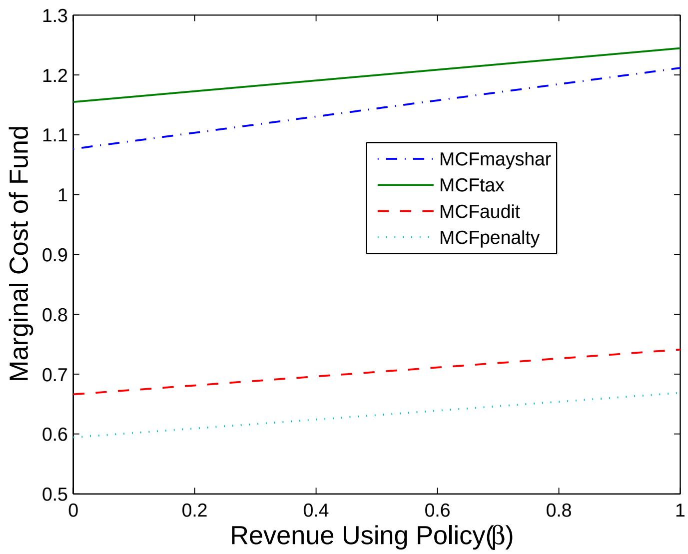
Figure 1.2: MCFs and marginal resource cost of tax enforcement



1.6 Conclusion

In a general equilibrium of tax evasion, we analytically derive three MCFs for the nonlinear tax, audit, and fine rates. Ignoring the tax evasion behavior will underestimate the $\mathrm{MCF_T}$. In a world with tax evasion, individuals could get more welfare loss from both tax distortion and uncertainty introduced by tax evasion. When calculating $\mathrm{MCF_T}$, taking the risk-bearing cost of evasion into account will give more accurate estimates. Thus, governments should consider the risk-bearing cost of tax evasion while deciding on publicly funded projects. Alm (1985) argues that tax evasion can cause another type of burden because of inefficient allocation of resources by taxpayers who try to evade. Even though the $\mathrm{MCF_T}$ estimates in this paper are always

Figure 1.3: MCFs and public spending policy



greater than in Mayshar (1991) and Stuart (1984), they are highly dependent on values assigned to audit and fine rate, however.

Governments can also use audit rate or fine rate in addition to tax rate as a policy tool. Considering efficiency cost of each policy is important when a government decides how to collect additional tax revenue. Our calculations show that, compared to a tax reform, an enforcement mechanism generates a lower MCF if the marginal resource cost of enforcement is low. However, when audit and fine elasticities of labor supply are negative and the marginal resource cost of using these policies is high, then the tax reform might have a lower MCF than the enforcement reform. In addition, it may not always be feasible to use enforcement policies in practice. For example, higher income individuals or some firms may lobby to government not to increase audit rate or penalty rate on tax evasion. Thus, using audit or fine rate might have some additional costs to a society in terms of effort and time spent in passing bills in congress. Our model does not cover this aspect of the economy. The magnitudes of MCFs are mainly determined by elasticities of labor supply and marginal costs of enforcement policies in our model. Our model suggests that when enforcement is costless or has a low cost and labor supply elasticities are positive, using enforcement as a policy tool is superior to a tax reform. Furthermore, when a government increase a tax rate, increasing enforcement neutralizes the extra distortion caused by the increased tax rate.

Chapter 2

Social Status, Conspicuous Consumption Levies, and Distortionary Taxation

2.1 Introduction

Tax system is very diverse in practice. Whereas the United States relies mainly on income taxation, many countries in the European Union weight differential commodity taxation. Further, developing countries usually impose differential taxes on luxury goods to prevent wasteful consumption. As in the view of Veblen (1899), people advertise their positions in a wealth hierarchy by consuming expensive products such as Rolex watches. Such individual behaviors set off a status-seeking game that wastefully allocates income for more "positional goods." Nonetheless, to seek higher status through conspicuous consumption, ones have much incentive to supply labor at a higher level, which can alleviate preexisting income-tax distortion. Accordingly, differential taxes on positional goods reduce this positive effect of conspicuous consumption on labor supply even though reallocating income for more "nonpositional goods." Therefore, the differential taxes generate two opposite consequences in a tax system.

Without arriving at definite conclusions and with being still open to debate even today, much has been written about the choice between differential commodity and income taxes for an optimal tax structure. The prominent work of Atkinson and Stiglitz (1976) and subsequent studies by Christiansen (1984) and Saez (2002) conclude that an optimal tax structure consists of only nonlinear income taxes without any commodity taxes under the condition in which utility is separable between commodities and leisure.[1] Moreover, Kaplow (2006) shows that commodity taxation is still undesirable even if the preexisting income taxes are not optimal as in reality. In other words, differential commodity taxes cannot supplement either optimal or not optimal nonlinear income taxes. However, all the previous studies ignore status effect in a form of externality that the consumption of positional goods carries. The differential taxes on positional goods in this case can function as a corrective tool for cutting down wasteful consumption from status-seeking race. In addition, the taxes also generate revenue that can be used to lessen preexisting income-tax distortion. Hence, the differential taxes on positional goods may supplement the nonlinear income taxes by generating a "double dividend" — not only a reduced conspicuous consumption, but also a less distortionary tax structure.

A related literature on environmental economics pays attention to optimal second-best environmental taxes when distortionary income taxes are required for a given revenue. Extending the basic point of Pigou (1920) to the second-best case, Sandmo (1975) ascertains that commodity taxes can be

[1]Commodity demands are unrelated to labor supply on that assumption.

used as an instrument for correcting resource misallocation that externalities bring about. Further, many papers on the 'double-dividend hypothesis' study the role of environmental taxes not only as a corrective tool for environmental externality but also as a revenue-raising device (see e.g., Bovenberg and de Mooij, 1994, Parry, 1995, 1997, Goulder et al., 1997, Parry et al., 1999, Fullerton and Metcalf, 2001). These studies show that the optimal tax is generally positive on a good that causes a negative externality. The "positional externality" studied in this paper is similar in that it entails a negative externality, but differs because that externality also provides an extra incentive to supply more labor. This second-best environmental tax literature also suggests that optimal taxes will be lower on goods that encourage labor supply (e.g., see Williams, 2002). Thus there are two offsetting effects on the optimal tax rate, unlike in the standard case considered in the environmental literature.[2]

The purpose of this paper is to explore optimal tax structure in the presence of status effect. I develop a game-theoretic model in which each individual with a different labor productivity unknown to the others engages in a status-seeking game, and government has a revenue requirement. Then I examine the welfare effect of a revenue-neutral shift in the tax-mix away from nonlinear income taxes towards positional-good taxes.

[2]Employing a status-seeking game, Frank (1985) as well as Hopkins and Kornienko (2004) show that individuals have relatively fewer expenditures on nonpositional goods for a given exogenous income. In this case, the offsetting effect on labor supply would not appear. However, this paper's model allows for endogenous labor supply and thus the two offsetting effects on the optimal tax are both present.

The contribution of this work is to provide economic rationale for practical use of differential taxes on positional goods which could be corrective and even revenue-raising instruments in a tax system. To correct the externality from conspicuous consumption, Hopkins and Kornienko (2004) suggest per-unit taxes positively associated with income levels, since individuals with relatively higher incomes spends money more on positional goods and, thus, generate the externality more. The corrective instrument that depends on income is too unrealistic to apply to the actual tax environment. In contrast, this paper employs differential commodity taxes which do not depend on the income levels, so that a government should be able to establish the real application. It still has the function of correcting the externality, however. Ireland (2001) finds only optimal income taxes, under an assumption that a tax authority cannot directly observe how individuals consume positional goods, and thus cannot levy taxes on conspicuous consumption. Thus, the optimal income taxes by itself acts as an instrument for both corrective and revenue-raising objectives. However, a government can know which types of commodities are visible and carry status effect in the real world. I present a government problem in which the tax authority choose an alternative tax system when given a set of possible tax instruments (i.e. not only income taxes but also commodity taxes). Finally, the results of Saez (2002) cannot give any specific guidelines for practical use of differential taxes on certain types of goods, because a government does not know enough about individual tastes of particular goods. However, more about wealthy individuals' tastes of certain kinds of commodities can be easily known. This analysis is based on conspicuous consumption in which individuals engage to flaunt their wealth. Since the rich spend relatively more on positional goods to enhance their social positions, this paper could provide useful insights to set differential commodity taxes on certain types of goods.

On the same separable-utility assumption as used in Atkinson and Stiglitz (1976), Christiansen (1984), and Saez (2002), I show that the revenue-neutral shift in the tax-mix away from nonlinear income taxes towards positional-good taxes enhances welfare. Hence, an optimal tax structure should have differential taxes on positional goods together with nonlinear income taxes. On the other hand, Atkinson and Stiglitz (1976), Christiansen (1984), and Saez (2002) insist that the differential commodity taxes cannot supplement the nonlinear income taxes and, thus, are necessary for the optimal tax structure. Furthermore, I find that differential taxes on positional goods are required to some extent, even if demands for positional goods are positively related to labor supply. In this case, Christiansen (1984) suggests negative commodity taxes or subsidies, however.

This paper is organized as follows. In the next section, I present a status-seeking game that can be used to examine an optimal tax system in the presence of status effect. Section 3 solves a two-stage optimization problem, explains individual behaviors in a symmetric Nash equilibrium, and gives an optimal income taxes without any commodity taxes. In section 4, I explore total welfare effect of the marginal revision in positional-good taxes. I offer concluding remarks in the final section.

2.2 Model environment

Consider an economy that has a unit measure of individuals who are identical in all respects, except work ability. A level of ability θ in some interval $\Theta := \left[\underline{\theta}, \bar{\theta}\right]$ with $\underline{\theta} \geq 0$ is given to an individual. The upper and lower bars stand for maximum and minimum levels, respectively. Each individual learns his or her own ability of work, whereas the others cannot know it. That is, one's work ability is unobservable and thus becomes private information. However, all of the individuals have common knowledge of the ability distribution in this economy, represented by a twice continuously differentiable distribution function $G(\cdot)$ on the ability set Θ, and $g(\theta) := \mathrm{d}G/\mathrm{d}\theta$ is the probability density function. Work ability is assumed to be equal to wage rate, since it implies labor productivity.

An individual who devotes his or her work hours l to the labor market at a given wage rate θ earns a gross income $i := \theta l$. The time endowment is normalized at one (hence, $l + L = 1$ with L denoting leisure). When a tax authority in this society sets an income tax schedule $T(i, s)$, disposable income becomes $z := i - T(i, s)$. The shift parameter s determines the shape of the income tax function. The individual takes this parameter as a given, since it is a choice variable of the tax authority. Now, disposable income is divided into two kinds of consumable goods, x and y, each of which can be distinguished from the other by some characteristics, and the budget constraint is

$$q_x x + q_y y = z \tag{2.1}$$

where $q_x := p_x + t_x$ and $q_y := p_y + t_y$. The tax rates and producer prices on goods are denoted by t and p, with each subscript indicating one of two goods. Hence, q means the after-tax commodity prices.

Using the terminology of Frank (1985), the above consumption goods are classified into two types, positional goods and nonpositional goods. The positional good x is so visible that it carries social status to certain individuals. The consumption of a positional good is referred to as conspicuous consumption. In contrast, the consumption of nonpositional good y does not affect one's status because others cannot directly observe it. Adopting the formulations of Frank (1985), Robson (1992), and Hopkins and Kornienko (2004), I assume that an individual's social status is determined not only by his or her consumption level of the positional good but also others' as follows:

$$R\left(x, F\left(\cdot\right)\right) := r + F\left(x\right) = r + \int_{\underline{x}}^{x} f\left(x'\right) \mathrm{d}x' \tag{2.2}$$

where $F\left(\cdot\right)$ is the distribution function of positional goods in this economy, and $f\left(\cdot\right)$ is its probability density function. Each individual has an incentive to consume a higher amount of the positional good and in turn seek a higher status than others, since the value of $F\left(x\right)$ is increasing in his or her own consumption level of x but decreasing in others'. The person with the lowest positional good level $\underline{x}$ earns the rank of r. Therefore, the parameter r, which is assumed to be positive or zero, becomes the lowest status in this society.

I assume that all individuals have the following identical utility function:

$$U\left(x, y, l, R\left(x, F\left(\cdot\right)\right)\right) := u\left(x, y, l\right) R\left(x, F\left(\cdot\right)\right) \tag{2.3}$$

where the conventional utility index $u\left(\cdot\right) \geq 0$ is a twice differentiable function which is strictly increasing and quasiconcave in two goods x and y, but strictly decreasing and quasiconvex in hours of work l. In eq. (2.3), the conventional utility function and the above status function enter multiplicatively into the utility function $U\left(\cdot\right)$. Hence, each individual faces a trade-off between a higher status and a lower direct utility when raising budget share of positional goods but reducing that of nonpositional goods.

In this society, the government has a revenue requirement. To maintain constituent welfare at a particular level, the government must finance a constant expenditure E by income taxation or commodity taxation. Then the government budget constraint becomes

$$\int_{\Theta} T\left(i, s\right) \mathrm{d}G + \int_{\Theta} \left(t_x x + t_y y\right) \mathrm{d}G = E \tag{2.4}$$

where I normalize the population size at unity.[3] Since the purpose of the present analysis is to investigate the welfare effect of a revenue-neutral shift in the tax mix away from income tax and toward tax on positional goods, I exploit the analytical method as presented in Christiansen (1984). When the government sets the optimal income taxes $T^*\left(i\right)$ for each i without any

[3] Note that $\int_{\Theta} \chi\left(\theta\right) \mathrm{d}G = \int_{\underline{\theta}}^{\bar{\theta}} \chi\left(\theta\right) g\left(\theta\right) \mathrm{d}\theta$ for any $\chi\left(\cdot\right)$ on Θ.

commodity taxes, the shift parameter s and some arbitrary function $\zeta(\cdot)$ can generally rewrite the income tax function as $T(i,s) := T^*(i) + s \cdot \zeta(i)$ and easily define any shift from the optimal income taxes. The value of s should be zero at the optimum.

I define a social welfare function $W(\cdot)$ that includes all possible redistributive tastes of the government as

$$W(q_x, q_y, s) := \int_{\Theta} \omega(\theta)\, u(x,y,l)\, R(x, F(\cdot))\, \mathrm{d}G \tag{2.5}$$

in which the function $\omega(\cdot)$ on Θ is such that $\int_{\Theta} \omega(\theta)\, \mathrm{d}\theta = 1$, and then $\omega(\theta)$ is the social weight on individuals with a wage rate $\theta \in \Theta$. Given a certain redistributive object, the level of social welfare varies according to the alternative tax system because individuals alter their consumption and labor supply behaviors. Thus, the government will design a tax structure that minimizes tax distortions.

2.3 Status-seeking game and income taxation

Before proceeding, it could be helpful to explain the timing of decisions by both individuals and government. As in Christiansen (1984) and Saez (2002), I decompose the individual maximization problem into two stages.[4] At the first stage, all individuals engage in the status-seeking game, and they noncooperatively decide their demands for positional and nonpositional goods

[4]The two-stage optimization is equivalent to the original one, even though this analysis is in the context of a simultaneous game.

— conditional to their hours of work. Then the individuals move on to the second stage and choose their hours of work — conditional to the income tax function that the government imposes. Embodying individual demands for two goods and labor supplies in its constraints, the government optimally chooses a tax function at the end of this section.

2.3.1 Noncooperative demands for goods

This subsection derives the conditional demands for positional and nonpositional goods when hours of work l are considered as fixed. Regarding the after-tax prices q_x and q_y on two consumption goods, hours of work l, and disposable income z as given exogenously at this stage, each individual chooses an amount of positional good x and a level of nonpositional good y to maximize the utility index in eq. (2.3), subject to the budget constraint in eq. (2.1), as follows:

$$\begin{aligned} &\max_{x,y}\; u\left(x,y,l\right)\left(r+F\left(x\right)\right) \\ &\text{s.t. } q_x x + q_y y = z. \end{aligned} \tag{2.6}$$

In eq. (2.6), the individual choice of positional good interrelates with those of the others by the distribution of positional goods in this society. Hence, the distribution function $F\left(\cdot\right)$ will be determined endogenously in the model.

Since they are identical in all respects but differ only in wage rate, all individuals decide a symmetric Nash equilibrium demand function for positional goods, such that $x = x\left(G\left(\theta\right)\right)$ for each original rank $G\left(\theta\right)$ in the exogenously given wage-rate hierarchy $[0,1]$. Assume for the moment that the equilibrium demand function $x(\cdot)$ is differentiable and strictly increasing. An individual with a relatively higher rank of wage rate has relatively more expenditure for positional goods in this society. Thus, the probability that an individual with an original wage-rate rank $G(\theta)$ consumes the positional good at a higher level than a randomly chosen individual with a rank $G(\theta')$ from the society is rewritten as

$$
\begin{aligned}
F(x) &= \mathrm{P}\left\{\theta' \in \Theta : x(G(\theta')) \leq x\right\} = \mathrm{P}\left\{\theta' \in \Theta : G(\theta') \leq x^{-1}(x)\right\} \\
&= \mathrm{P}\left\{\theta' \in \Theta : \theta' \leq G^{-1}\left(x^{-1}(x)\right)\right\} = G\left(G^{-1}\left(x^{-1}(x)\right)\right) \\
&= x^{-1}(x) \ \text{ for any } x
\end{aligned}
\tag{2.7}
$$

where the superscript $^{-1}$ indicates the inverse of a given function. Eq. (2.7) equates the distribution function $F(\cdot)$ of positional goods with the inverse $x^{-1}(\cdot)$ of an equilibrium demand function for positional goods.

Substituting eq. (2.7) into the individual maximization problem (2.6), we have the following Lagrange expression:

$$
\mathfrak{L}(x, y, \mu; q_x, q_y, l, z) = u(x, y, l)\left(r + x^{-1}(x)\right) + \mu(z - q_x x - q_y y). \tag{2.8}
$$

Setting the partial derivatives of the Lagrange (2.8) equal to zero yields the first-order conditions:

$$
\mathfrak{L}_x = u_x(x, y, l)\left(r + x^{-1}(x)\right) + u(x, y, l)\frac{\partial\left(x^{-1}(x)\right)}{\partial x} - \mu q_x = 0, \tag{2.9}
$$

$$
\mathfrak{L}_y = u_y(x, y, l)\left(r + x^{-1}(x)\right) - \mu q_y = 0, \tag{2.10}
$$

$$
\mathfrak{L}_\mu = 0 : z - q_x x - q_y y = 0.
$$

Note that $x = x(G(\theta))$ for each $G(\theta) \in [0,1]$ in a symmetric equilibrium. Then eq. (2.7) gives the following equalities:

$$F(x) = x^{-1}(x) = G(\theta) \text{ for each } \theta \in \Theta \text{ in equilibrium.} \tag{2.11}$$

Hence, each individual rank in the distribution of positional goods becomes equal to the original rank in the wage-rate distribution in this society. That is, once all individuals noncooperatively determine their demands for positional goods, they learn that their status is at the same position as in the exogenously given wage-rate hierarchy. Plugging eq. (2.11) into eqs. (2.9) and (2.10) can rewrite the first order conditions at the noncooperative equilibrium as

$$\mathfrak{L}_x = u_x(x,y,l)(r + G(\theta)) + u(x,y,l)\frac{\partial G(\theta)}{\partial x} - \mu q_x = 0, \tag{2.12}$$

$$\mathfrak{L}_y = u_y(x,y,l)(r + G(\theta)) - \mu q_y = 0, \tag{2.13}$$

$$\mathfrak{L}_\mu = 0 : z - q_x x - q_y y = 0. \tag{2.14}$$

Using eqs. (2.12) and (2.13), we arrive at the tangency condition:

$$\frac{u_x(x,y,l)}{u_y(x,y,l)} + \frac{u(x,y,l)}{u_y(x,y,l)(r + G(\theta))}\frac{\partial G(\theta)}{\partial x} = \frac{q_x}{q_y}. \tag{2.15}$$

The first term in eq. (2.15) is the marginal rate of substitution between x and y for the standard maximization problem of the consumer. But the condition includes another term that implies an additional marginal return to the consumption of x, since the positional good carries social status. Each individual has relatively less expenditure for nonpositional goods and relatively more for positional goods due to this additional positive return. Such noncooperative equilibrium demands for goods are inefficient in terms of conventional utility.

From the collective point of view, the indirect return to conspicuous consumption is undesirable in eq. (2.15). Following the cooperative case that Frank (1985) formulates in this context, I pose the maximization problem of each cooperating individual as

$$\max_{\tilde{x},\tilde{y}} u\left(\tilde{x},\tilde{y},\tilde{l}\right)(r+G(\theta)) \qquad (2.16)$$
$$\text{s.t.} q_x\tilde{x}+q_y\tilde{y}=\tilde{z}$$

where the tilde $\sim$ on a variable stands for cooperative case. In optimizing utility, each individual takes status as the originally given position in the wage-rate distribution. Hence, cooperative utility maximization in eq. (2.16) yields the usual tangency conditions:

$$\frac{u_x\left(\tilde{x},\tilde{y},\tilde{l}\right)}{u_y\left(\tilde{x},\tilde{y},\tilde{l}\right)}=\frac{q_x}{q_y}. \qquad (2.17)$$

Eq. (2.17) eliminates the spurious return that eq. (2.15) takes in the noncooperative case. The marginal rate of substitution between x and y is equal to the relative price q_x/q_y. Thus, the cooperative equilibrium demands for goods are efficient in terms of conventional utility.

Now, replacing for y using eq. (2.14), the tangency condition (2.15) in the noncooperative case gives a first-order ordinary differential equation:

$$\frac{\partial x}{\partial G(\theta)}=\frac{u\left(x,\frac{z}{q_y}-\frac{q_x}{q_y}x,l\right)}{(r+G(\theta))\left(\frac{q_x}{q_y}u_y\left(x,\frac{z}{q_y}-\frac{q_x}{q_y}x,l\right)-u_x\left(x,\frac{z}{q_y}-\frac{q_x}{q_y}x,l\right)\right)}$$
$$=\xi\left(x,G(\theta);q_x,q_y,z,l\right). \qquad (2.18)$$

Using the cooperative demand for positional good $\tilde{x}$, we have the boundary condition for the differential equation (2.18) as follows:

$$x(0) = \begin{cases} \underline{\theta}/q_x & \text{if } r = 0 \\ \tilde{x} \text{ with } G\left(\underline{\theta}\right) = 0 & \text{if } r > 0. \end{cases} \tag{2.19}$$

The poorest individuals with rank $G\left(\underline{\theta}\right) = 0$ spend all income on the positional good, if the bottom of status r is zero in this society. That is the only way in which an individual increases utility, since that person's equilibrium utility $u\left(x\left(0\right), y\left(0\right), l\right) G\left(\underline{\theta}\right)$ is zero. On the other hand when r is positive, an individual demand for positional goods is the same as in the cooperative case. That is, an individual consumes the positional good without any interest in status seeking.

Given the values of q_x, q_y, z, and l, the differential equation (2.18) with the boundary condition (2.19) forms the equilibrium demand for positional goods that is differentiable and strictly increasing in wage-rate ranking $G\left(\theta\right)$.[5] Thus, the solution to the differential equation, together with eqs. (2.13) and (2.14), yields the demands for two goods and the Lagrangian multiplier in the following forms:

$$x = x\left(q_x, q_y, z, l, G\left(\theta\right)\right) \tag{2.20}$$

$$y = y\left(q_x, q_y, z, l, G\left(\theta\right)\right) \tag{2.21}$$

$$\mu = \mu\left(q_x, q_y, z, l, G\left(\theta\right)\right). \tag{2.22}$$

[5]This paper does not focus on the formal proof. See Hopkins and Kornienko (2004) for details in the context of the status-seeking game, however.

Consequently, we write the corresponding indirect utility function, conditional on hours of work l, as

$$\begin{aligned} v\left(q_x, q_y, z, l, G\left(\theta\right)\right)\left(r + G\left(\theta\right)\right) =& u\left(x\left(q_x, q_y, z, l, G\left(\theta\right)\right), y\left(q_x, q_y, z, l, G\left(\theta\right)\right), l\right) \\ & \times \left(r + x^{-1}\left(x\left(q_x, q_y, z, l, G\left(\theta\right)\right)\right)\right) \end{aligned} \tag{2.23}$$

where the conventional indirect utility function is written as $v(q_x, q_y, z, l, G(\theta))$ $:= u(x, y, l)$, and $r + x^{-1}(x) = r + G(\theta)$ in equilibrium. Furthermore, the envelope theorem gives the following properties:

$$v_{q_x}\left(r + G\left(\theta\right)\right) = -\mu x \tag{2.24}$$

$$v_{q_y}\left(r + G\left(\theta\right)\right) = -\mu y \tag{2.25}$$

$$v_z\left(r + G\left(\theta\right)\right) = \mu \tag{2.26}$$

$$v_l = u_l \tag{2.27}$$

$$v_{G(\theta)}\left(r + G\left(\theta\right)\right) + v = 0 \tag{2.28}$$

in which a subscript on a function stands for a partial derivative with respect to it.

2.3.2 Labor supplies

In this subsection, second-stage optimization decides the hours of work l that have been treated as fixed until now. Also, gross and disposable incomes $i = \theta l$ and $z = i - T\left(i, s\right)$ are determined as a consequence. Regarding after-tax prices q_x and q_y on two consumption goods, shift parameter s in the income tax function, and wage rate θ as exogenously given, each individual chooses hours of work l to maximize the indirect utility index in eq. (2.23) as follows:

$$\max_{l} v\left(q_x, q_y, z, l, G\left(\theta\right)\right)\left(r + G\left(\theta\right)\right) \tag{2.29}$$
$$\text{s.t. } z = i - T\left(i, s\right)$$
$$i = \theta l.$$

This optimization reads the first-order condition as

$$\begin{aligned}\frac{\mathrm{d}v}{\mathrm{d}l} =& \left[v_z\left(q_x, q_y, z, l, G\left(\theta\right)\right)\left(1 - T_i\left(i, s\right)\right)\theta + v_l\left(q_x, q_y, z, l, G\left(\theta\right)\right)\right] \\ &\times\left(r + G\left(\theta\right)\right) = 0,\end{aligned} \tag{2.30}$$

and the second-order condition as $\mathrm{d}^2 v/\mathrm{d}l^2 < 0$.

To investigate incentives to labor supplies in this status-seeking game, we have two tangency conditions for the noncooperative and cooperative cases as follows:[6]

$$-\frac{u_x\left(x, y, l\right)}{u_l\left(x, y, l\right)} - \frac{u\left(x, y, l\right)}{u_l\left(r + G\left(\theta\right)\right)}\frac{\partial G\left(\theta\right)}{\partial x} = \frac{q_x}{\left(1 - T_i\left(i, s\right)\right)\theta}, \tag{2.31}$$

$$-\frac{u_x\left(\tilde{x}, \tilde{y}, \tilde{l}\right)}{u_l\left(\tilde{x}, \tilde{y}, \tilde{l}\right)} = \frac{q_x}{\left(1 - T_i\left(\tilde{\imath}, s\right)\right)\theta}. \tag{2.32}$$

In the noncooperative case (2.31), the marginal rate of substitution between the positional good x and leisure L $(= 1 - l)$ is less than the relative price $q_x/\left(1 - T_i\left(i, s\right)\right)\theta$. In contrast, the cooperative case (2.32) equates the marginal rate of substitution between $\tilde{x}$ and $\tilde{L}$ with $q_x/\left(1 - T_i\left(\tilde{\imath}, s\right)\right)\theta$. If an individual

[6]First, substitute eqs. (2.26) and (2.27) into eq. (2.30) to have $-u_l(r + G(\theta)) = \mu(1 - T_i(i, s))\theta$. Using this equation together with eq. (2.12), we arrive at eq. (2.31).

with wage rate θ faces the same marginal tax rate ($T_i(i, s) = T_i(\tilde{\imath}, s)$), and consumes both goods at equal levels in both cases ($x = \tilde{x}$ and $y = \tilde{y}$), then the marginal rate of substitution is less in the noncooperative case than in the cooperative one due to the positive additional return to positional goods. Thus, $L < \tilde{L}$ (or $l > \tilde{l}$), because the price of leisure is relatively more expensive in the noncooperative case. That is, seeking status through conspicuous consumption provides incentives to labor supplies in a society.

From the first-order condition (2.30), we obtain work hours and disposable income in the following forms:[7]

$$l = l(q_x, q_y, s, \theta) \tag{2.33}$$

$$z = z(q_x, q_y, s, \theta) \tag{2.34}$$

Plugging eqs. (2.33) and (2.34) into eq. (2.23) rewrites the indirect utility function as

$$\begin{aligned} &V(q_x, q_y, s, \theta)(r + G(\theta)) \\ &= v(q_x, q_y, z(q_x, q_y, s, \theta), l(q_x, q_y, s, \theta), G(\theta))(r + G(\theta)) \end{aligned} \tag{2.35}$$

where $V(q_x, q_y, s, \theta) := v(q_x, q_y, z, l, G(\theta))$. Note that given q_x, q_y, and s, the indirect utility in eq. (2.35) is a function of wage rate θ only.

[7]The hours of work l actually depends on the rank $G(\theta)$ in the distribution of wage rates: $l = l'(q_x, q_y, s, \theta, G(\theta))$. Since $l'(q_x, q_y, s, \theta, G(\theta)) = l(q_x, q_y, s, \theta)$, we can arrive at eq. (2.33), however.

2.3.3 Optimal income taxes

Since the analysis investigates the welfare effect of a marginal increase in tax on positional goods at the optimal level of income tax $T^*(i)$, characterizing the shape of the optimal income tax in detail is not of concern. Thus, this subsection focuses only on the condition that characterizes the optimal income tax without any commodity taxes, and the parametric optimization is enough to accomplish that. Thus, the government chooses shift parameter s in the income tax function to optimize social welfare in eq. (2.5), subject to the government budget constraint in eq. (2.4), as follows:

$$\begin{aligned} &\max_s W(q_x, q_y, s) \\ &\text{s.t.} \int_\Theta T(i,s)\,\mathrm{d}G + \int_\Theta (t_x x + t_y y)\,\mathrm{d}G = E \\ &\quad t_x = t_y = 0. \end{aligned} \tag{2.36}$$

This government optimization yields the first-order condition:

$$\frac{W_s(q_x, q_y, s)}{\lambda} + \int_\Theta \left(T_i(i,s)\,\theta l_s + T_s(i,s)\right)\mathrm{d}G = 0 \tag{2.37}$$

where the Lagrange multiplier $\lambda = -W_E$ is evaluated at the optimum. Hence, we get the condition (2.37) that characterizes the optimal income tax $T^*(i)$ for each i, if the shift parameter s is zero.

2.4 The welfare effect of conspicuous consumption taxes

To explore the total welfare effect of a revenue-neutral change in the tax mix away from income tax and toward tax on conspicuous consumption, the economy is assumed to be in such a state that the government sets an income tax optimally and does not levy on consumable goods. That is, this analysis starts at an initial equilibrium with an existing optimal income tax ($s = 0$), but without any commodity taxes ($t_x = t_y = 0$), and then introduces a small tax $\mathrm{d}t_x$ on conspicuous consumption. Differentiating the social welfare function (2.5) totally and substituting the first-order condition (2.37) that characterizes the optimal income tax, we have

$$\begin{aligned} \frac{\mathrm{d}W}{\lambda} &= \frac{W_{q_x}}{\lambda}\mathrm{d}t_x + \frac{W_s}{\lambda}\mathrm{d}s \\ &= \frac{W_{q_x}}{\lambda}\mathrm{d}t_x + \left[-\int_\Theta \left(T_i\theta l_s + T_s\right)\mathrm{d}G\right]\mathrm{d}s. \end{aligned} \tag{2.38}$$

The left-hand side of eq. (2.38) is the dollar value of the change in social welfare ($\mathrm{d}W/\lambda$). Since I explore a change in the tax mix, I assume that the government does not change the public expenditure ($\mathrm{d}E = 0$). Differentiating the government budget constraint (2.4) totally and evaluating s, t_x, and t_y with zero in turn yield

$$\left[\int_\Theta \left(T_i\theta l_s + T_s\right)\mathrm{d}G\right]\mathrm{d}s + \left[\int_\Theta T_i\theta l_{t_x}\mathrm{d}G + \int_\Theta x\mathrm{d}G\right]\mathrm{d}t_x = 0. \tag{2.39}$$

Replacing eq. (2.39) into eq. (2.38) and subtracting eq. (2.37) from this give

$$\frac{\mathrm{d}W}{\lambda\mathrm{d}t_x} = \frac{W_{q_x} - W_s}{\lambda} + \int_\Theta \left(x - T_s\right)\mathrm{d}G + \int_\Theta T_i\theta\left(l_{t_x} - l_s\right)\mathrm{d}G. \tag{2.40}$$

The left-hand side of eq. (2.40) is the total welfare change in terms of dollar value when the government imposes a small tax change $\mathrm{d}t_x$ on conspicuous consumption. The first term implies the change in tax burden, and the second and third terms represent the change in total revenue on the right-hand side of eq. (2.40). Hence, the total welfare change is divided into these two terms.

The next step is to evaluate the change in tax burden on the right-hand side of eq. (2.40). From eq. (2.35), the social welfare function is given as

$$W(q_x, q_y, s) = \int_\Theta \omega(\theta) V(q_x, q_y, s, \theta)(r + G(\theta)) \, \mathrm{d}G. \tag{2.41}$$

Differentiating the social welfare function (2.41) partially with respect to q_x and s, we get

$$\begin{aligned} \frac{W_{q_x} - W_s}{\lambda} &= \frac{1}{\lambda} \int_\Theta \omega(\theta)(V_{q_x} - V_s)(r + G(\theta)) \, \mathrm{d}G \\ &= \int_\Theta \left(\frac{\omega(\theta) v_z (r + G(\theta))}{\lambda} \right) (T_s - x) \, \mathrm{d}G \end{aligned} \tag{2.42}$$

which is applied with two equalities that $V_{q_x} = v_{q_x} = -xv_z$ and $V_s = -v_z T_s$ from eqs. (2.24), (2.26), and (2.30). Plugging eq. (2.42) into eq. (2.40) rewrites the total welfare change as

$$\begin{aligned} \frac{\mathrm{d}W}{\lambda \mathrm{d}t_x} &= \int_\Theta (T_s - x) \left(\frac{\omega(\theta) v_z (r + G(\theta))}{\lambda} - 1 \right) \mathrm{d}G \\ &\quad + \int_\Theta T_i \theta (l_{t_x} - l_s) \, \mathrm{d}G. \end{aligned} \tag{2.43}$$

Following the analytical method that Christiansen (1984) uses, I define the marginal shift as $T_s(i, s) = \zeta(i) = x$. In a symmetric Nash equilibrium, each individual choice ultimately depends on the wage rate θ only. Therefore, the equilibrium demand for positional goods is described as $x = x(\theta)$ for each $\theta \in \Theta$. Since gross income i is strictly increasing in θ, we can rewrite the wage rate as $\theta = \theta(i)$ for all i. Substituting this function of wage rate into $x(\theta)$ yields the equilibrium demand for positional goods as a function of income — that is, $x(\theta) = x(\theta(i))$. Hence, the income tax function $T(i, s) = T^*(i) + s \cdot x(\theta(i))$ is well defined for all levels of gross income i, since the equilibrium demand for positional goods x is expressed for all levels of gross income i. The first term vanishes out in eq. (2.43). Then the total welfare change becomes

$$\frac{\mathrm{d}W}{\lambda \mathrm{d}t_x} = \int_{\Theta} T_i \theta \left(l_{t_x} - l_s\right) \mathrm{d}G. \tag{2.44}$$

Since the per-hour income tax $T_i\theta$ is positive, the sign of the term $(l_{t_x} - l_s)$ determines the total welfare effect of marginal tax $\mathrm{d}t_x$ on positional goods.

The final step is to assess the sign of the term $(l_{t_x} - l_s)$ in the total welfare change (2.44). Differentiating the first-order condition for hours of work l in eq. (2.30) with respect to t_x and s, we get

$$\frac{\partial}{\partial t_x}\left(\frac{\mathrm{d}v}{\mathrm{d}l}\right) = \left(\frac{\mathrm{d}^2 v}{\mathrm{d}l^2}\right) l_{t_x} + \left(v_{zq_x}\left(1 - T_i\right)\theta + v_{lq_x}\right)\left(r + G(\theta)\right) = 0 \tag{2.45}$$

$$\begin{aligned}\frac{\partial}{\partial s}\left(\frac{\mathrm{d}v}{\mathrm{d}l}\right) = \left(\frac{\mathrm{d}^2 v}{\mathrm{d}l^2}\right) l_s &+ \left(v_{zz}\left(-T_s\right)\left(1 - T_i\right)\theta + v_z\left(-T_{is}\right)\theta + v_{lz}\left(-T_s\right)\right) \\ &\times \left(r + G(\theta)\right) = 0.\end{aligned} \tag{2.46}$$

Subtracting eq. (2.46) from eq. (2.45), we have the difference between the changes in work hours, with respect to the tax on positional goods and the shift parameter in the income tax function, as follows:

$$\begin{aligned} l_{t_x} - l_s = &\left(-\frac{r + G(\theta)}{\mathrm{d}^2 v/\mathrm{d}l^2}\right) \\ &\times \left(\left(v_{zq_x} + v_{zz}T_s\right)\left(1 - T_i\right)\theta + v_{lq_x} + v_{lz}T_s + v_z T_{is}\theta\right). \end{aligned} \tag{2.47}$$

The defined marginal shift $T_s(i,s) = x$, together with eqs. (2.24) and (2.26), gives the following relationship:

$$T_s(i,s) = x(q_x, q_y, z, l, G(\theta)) = -\frac{v_{q_x}(q_x, q_y, z, l, G(\theta))}{v_z(q_x, q_y, z, l, G(\theta))}. \tag{2.48}$$

Plugging the above equalities (2.48) into eq. (2.47) and dividing this result by v_z, we have

$$\begin{aligned} l_{t_x} - l_s = &\left(-\frac{v_z(r + G(\theta))}{\mathrm{d}^2 v/\mathrm{d}l^2}\right) \\ &\times \left(\left(\frac{v_{zq_x}}{v_z} - \frac{v_{zz}v_{q_x}}{v_z^2}\right)(1 - T_i)\,\theta + \left(\frac{v_{lq_x}}{v_z} - \frac{v_{lz}v_{q_x}}{v_z^2}\right) + T_{is}\theta\right). \end{aligned} \tag{2.49}$$

Differentiating the demand for positional good x in eq. (2.48) partially with respect to the disposable income z and the hours of work l, we then have

$$x_z = -\left(\frac{v_{q_x z}}{v_z} - \frac{v_{q_x}v_{zz}}{v_z^2}\right) \text{ and } x_l = -\left(\frac{v_{q_x l}}{v_z} - \frac{v_{q_x}v_{zl}}{v_z^2}\right). \tag{2.50}$$

Using eq. (2.50) and the fact that $T_{is} = T_{si} = \partial x/\partial i$, we rewrite the equation (2.49) as

$$l_{t_x} - l_s = \left(-\frac{v_z(r + G(\theta))}{\mathrm{d}^2 v/\mathrm{d}l^2}\right)\left(\frac{\partial x}{\partial i}\theta - x_z(1 - T_i)\,\theta - x_l\right). \tag{2.51}$$

Since the wage rate θ can be expressed as a function of gross income i, disposable income $z = z(\theta)$ and hours of work $l = l(\theta)$ are also functions of gross income i. This fact yields the derivative of the equilibrium demand for positional goods with respect to gross income i as

$$\frac{\partial x}{\partial i} = x_z(1 - T_i) + x_l\frac{\partial l}{\partial i} + x_{G(\theta)}g(\theta)\frac{\mathrm{d}\theta}{\mathrm{d}i}. \tag{2.52}$$

Replacing eq. (2.52) into eq. (2.51) and using the relationship that $1-\theta\partial l/\partial i = l\mathrm{d}\theta/\mathrm{d}i > 0$ from the definition of gross income $i = \theta l$, we arrive at

$$l_{t_x} - l_s = \left(-\frac{v_z\left(r + G\left(\theta\right)\right)}{\mathrm{d}^2 v/\mathrm{d}l^2}\right)\left(x_{G(\theta)} g\left(\theta\right)\left(\frac{\theta}{l}\right) - x_l\right)\left(\frac{l\mathrm{d}\theta}{\mathrm{d}i}\right). \tag{2.53}$$

Finally, using eq. (2.53), we rewrite the total welfare change (2.44) as

$$\frac{\mathrm{d}W}{\lambda \mathrm{d}t_x} = \int_{\Theta} T^* \left(-\frac{v_z\left(r + G\left(\theta\right)\right)}{\mathrm{d}^2 v/\mathrm{d}l^2}\right)\left(x_{G(\theta)} g\left(\theta\right)\cdot\frac{\theta}{l} - x_l\right)\left(\frac{\mathrm{d}\theta}{\mathrm{d}i}\right)\mathrm{d}G \tag{2.54}$$

where T^* is an optimal tax (i.e., $T\left(i, s\right) = T^*\left(i\right)$ for $s = 0$). Note that all the terms T^*, $\left(-\frac{v_z(r+G(\theta))}{\mathrm{d}^2 v/\mathrm{d}l^2}\right)$, and $\mathrm{d}\theta/\mathrm{d}i$ are positive in eq. (2.54). Moreover, the term $x_{G(\theta)} g\left(\theta\right)\left(\theta/l\right)$ is also positive, since the marginal effect of an increase in original wage-rate rank on demand for positional goods is always positive ($x_{G(\theta)} > 0$). Therefore, the total welfare change becomes positive, if the marginal effect of an increase in work hours on demand for positional goods is zero ($x_l = 0$). This case is true when the conventional utility function $u\left(x, y, l\right)$ is weakly separable between hours of work l and demand for positional good x.[8] If the demand for a positional good is unrelated to hours of work, then the shift in tax mix away from the income tax and toward the tax on positional goods increases social welfare. This statement implies that the tax on a positional good is desirable even when the preexisting income tax is optimal. In other words, both a differential tax on conspicuous consumption goods and an income tax constitute the optimal tax system in the presence of a status effect. Even if the demand for positional goods is positively related to

[8]See Christiansen (1984) for the proof.

hours of work (i.e., $x_l > 0$), the differential tax on a conspicuous consumption good can supplement the income tax to some extent.

2.5 Conclusion

The previous section observes a positive welfare effect when a government conducts a small change in conspicuous consumption tax under its revenue requirement. With a conventional utility function separable between positional goods and hours of work, I infer that the optimal tax system needs a supplementary tax on positional goods together with an income tax. The result is based on the assumption that the preexisting income tax is at an optimal level. Kaplow (2006) notes that the preexisting income tax may not be optimal in reality. Then he shows that commodity taxation is undesirable, even in this case. Commodity taxation would still be desirable with consumption externality, however, even if the income tax is not at an optimal level. This analysis identifies that the optimal income tax cannot correct the distortion from status-seeking behaviors by itself. Hence, a suboptimal income tax could leave the distortion at a higher level than an optimal one. In turn, the result of this paper would be consistent even when the preexisting income tax is not optimal.

I have demonstrated that a marginal tax increase on positional goods enhances each individual's conventional utility for any distribution of wage rates. Generally, individuals with the same wage rates have different utility levels in two societies that differ in the distribution of wage rates, however, since the equilibrium demand for positional goods varies according to the shape of the distribution. Thus, the alternative distributions of wage rates in two societies generate different social welfare effects of the marginal tax on positional goods. A comparison of social welfare effects in two societies would be meaningful for future research. This comparative statics could provide insight into the change in differential commodity tax, either as a society achieves higher levels of economic growth or as the constituents in the society become more equal in income.

Chapter 3

Capital Mobility, Growth, and Environment: "Race to the Bottom"

3.1 Introduction

Compared to the past, economic interdependence has gradually increased in recent years. The stock of capital as a factor of production (e.g., factory or machinery) does not need to be invested in a particular region. Many regional economies have achieved successful economic growth with a higher capital mobility. But, many environmentalists are concerned that the regions with a rapid growth rate experience many localized environmental externalities. An example is deforestation. If a higher capital mobility generates severe interregional competition, less stringent local standards could deteriorate the local environments. However, the increasing capital mobility might give both a higher economic growth and a better environment, if each locality is concerned with its environmental externality and sets an environmental measure according to its own interest. Therefore, ones may be somewhat confused about how the increasing economic integration affects the local economic development and environment.

The literature on local public finance and environmental economics gives two opposite conclusions on the "race to the bottom" in environmental standards. This hypothesis states that severe economic competition among local authorities will result in lower levels of environmental quality. Thus, federal government intervention is necessary to preserve local environments. Markusen et al. (1995) show that noncooperative behaviors of two regions generate welfare loss in a model with an endogenous plant location. On the other hand, it is also argued that each local jurisdiction could achieve an efficient environmental regulation by itself. Using a simple static model with interjurisdictional competition, Oates and Schwab (1988) contend that the local setting of environmental standards is globally optimal in the jurisdictions homogenous in workers.

The purpose of this paper is to examine how the increasing capital mobility impacts regional economic growth and environment. To answer the question, I develop an endogenous growth model in which each local government competes against the others to induce imperfectly mobile stock of capital into its region. Then I form a conclusion on the hypothesis above. Comparing three alternative policy systems between a federation and local jurisdictions, this paper presents the welfare implication, and it suggests which policy structure is better for regional development and environment preservation with the increasing capital mobility.

This paper contributes to the literature on interjurisdictional competition and growth theory as follows. It extends the issues of fiscal or regulatory policy competition to a dynamic framework. It develops an endogenous growth model with an imperfect capital mobility and local environments. Most previous studies employed static models to address the issues of decentralized environmental regulations. Hence, they do not consider dynamic features of capital as a stock variable. As the stock of capital accumulates over time, the amount of pollution with no abatement activities grows at a positive rate. For a local economy to have a perpetual growth with a stable level of environmental quality, some forms of abatement activities should be present.[1] This paper uses a public expenditure as the abatement activity as in Smulders and Gradus (1996). Also, the accumulated stock of capital creates a positive externality on productivity growth in regional economies. In a particular region into which relatively more amounts of capital stock are induced, the local residents provide more productive labor supply to local production because of learning effects from investment activities of capital owners as in Arrow (1962). This technological change on labor productivity is incorporated in the model of this paper. Second, following the formulation of capital mobility as in Rauscher (2005), this paper uses the full range of capital mobility such that the stock of capital gets from perfectly immobile to perfect mobile. The scope of previous literature is limited to distortion-inducing or efficiency-enhancing arguments with a fixed capital mobility as in Oates and Schwab (1988) and Kunce and Shogren (2005). However, this adoption of capital mobility allows

[1]The literature on the endogenous growth and environmental economics has addressed how an economy can achieve sustainable growth with a stable level of environmental quality. Bovenberg and Smulders (1995) develop a two sector endogenous growth model with pollution-augmenting technology that helps the pollution be used more effectively over time.

us to analyze the impact of amalgamation on local policy variables, as if the economic integration had been increased. Moreover, all the federal and local policy variables are endogenized in this model. In order to look at the implication of distortionary tax, the previous papers usually specify the policy instruments of an upper level of government as exogenous as in Zodrow and Mieszkowski (1986) and Rauscher (2005). But, an upper level of government could respond to the policies set by the lower levels of government in the real policy system, as is done in the United States.

For a given level of capital mobility, the local jurisdictions with a full range of local policies (i.e. no federal intervention) achieve a sustainable economic development. An increase in capital mobility generates "tax importing" due to which each locality experiences a higher growth rate and more degraded environment. That is, the increasing mobility dampens the capital tax and transfers the burden of pollution abatement to the locality. The capital tax rate is negative and the local environment is deteriorated completely at the perfect capital mobility. This finding supports the hypothesis of "race to the bottom" in environmental standards. This paper also identifies that the increasing capital mobility raises a higher growth rate, but it reduces the overall welfare of residents. To avoid the cut-throat competition and preserve regional environments, an upper level of government must intervene to save the single jurisdictions.[2] An uniform environmental standard and a requirement of lump sum transfer (or tax) are considered as the federal interventions in this model.

[2]Cumberland (1979, 1981) suggests the uniformly minimum environmental standards.

Both of two optimal interventions improve the residents' welfare. The optimal uniform environmental standard is independent of any capital mobility, so that it prevents local environment from degradation by the increasing capital mobility (or severe regional competition). However, the optimal transfer (or tax) requirement degrades the environmental quality more if the resident's elasticity of intertemporal substitution is less than one, even though it secures a consumption level for the local residents against the increasing capital mobility.

The paper is organized as follows. In the next section, I present an endogenous growth model with interjurisdictional fiscal competition. Section 3 employs majority voting to solve for local decisions and then examines whether the local jurisdictions "race to the bottom or the top" in environmental standards. In section 4, two federal interventions are endogenized in the model. I investigate how the optimal interventions affect the local environmental quality and economic growth. The final section concludes and suggests future research.

3.2 The model

Assume that a federation consists of infinitely small identical jurisdictions, and each local jurisdiction has many atomistic profit-maximizing firms which take the role of local production. In the model, the federation is represented as a unit real plane $[0,1] \times [0,1]$ in which a firm i of a jurisdiction j is expressed as one specific point $(i,j) \in [0,1] \times [0,1]$. In each jurisdiction, two types of agents, called capitalist and worker, live on the infinite horizon. The members in each group are homogenous and large in number.[3] At each time, the capitalists can save but cannot work, while the workers consume all the earned incomes with no saving.[4] As local production factors, the capital stock that is a forgone consumption is imperfectly mobile, but the labor is immobile across jurisdictions.

3.2.1 Local environmental quality

The capital stock K_j invested in a local jurisdiction j generates pollution P_j through a production process. The pollution as an externality is 'bad' for the residents in the jurisdiction.[5] If a higher level of capital is located in the jurisdiction, the residents consume a lower level of local environmental quality. However, the local pollution can be reduced by public abatement activities G_j which is financed with a tax on capital by the authority of the jurisdiction. If the jurisdiction decides on more public expenditure for abatement activities, the capital stock induced in the jurisdiction deteriorates the local environment less. Thus, the local environmental quality of jurisdiction j at each time is

[3] Here, the workers are homogenous as wage earners. However, the paper extends the model to include non-wage workers whose circumstances and interests are different from those of wage workers.

[4] Judd (1985), Lejour and Verbon (1997), and Rauscher (2005) make this assumption.

[5] Local residents have disutility from pollution as a negative externality, so that the pollution is incorporated into the utility function. We could include the negative externality in the production function as well as the utility function, however. The pollution could have a negative effect on production if lower environmental quality creates less productivity as in Smulders and Gradus (1996) and Bovenberg and Smulders (1995). However, this aspect does not make any significant difference, even though this analysis does not consider it formally.

represented as the following pollution function

$$P_j = \left(\frac{K_j}{G_j}\right)^{\chi}, \tag{3.1}$$

where χ is a positive elasticity of pollution with respect to the capital-abatement ratio.[6] In eq. (3.1), the pollution is increasing in attracted stock of capital and decreasing in public abatement activities in jurisdiction j: $\partial P_j/\partial K_j > 0$, $\partial P_j/\partial G_j < 0$. Assume further that a polluting emission generated in one jurisdiction doesn't have a spillover effect on another. Therefore, the local environment is modeled as a purely public good that can be consumed within a particular jurisdiction.

3.2.2 Production technology

At each point in time $t \in [0, \infty)$, many atomistic profit-maximizing firms take the role of local production.[7] In order to produce a private good Y_{ij} that is sold in the national markets, each firm i in jurisdiction j employs stock of capital K_{ij} and labor L_{ij} and use the following form of production technology

$$Y_{ij} = Y(K_{ij}, L_{ij}, K_j) = AK_{ij}^{\alpha} L_{ij}^{1-\alpha} K_j^{1-\alpha}, \tag{3.2}$$

[6] The public expenditure G_j as pollution abatement activities as well as the pollution P_j are modeled as flow variables, e.g. filters used up within one period. Hence, in order to preserve a stable level of environmental quality, the public goods should be provided in each period. Modeling pollution or public abatement activities as stock values make this analysis difficult without critical different results along a balanced growth path.

[7] In order to reduce complication and save simplicity, the model omits the notation of time t on variable.

which exhibits conventional constant returns to scale in two factors of production, capital stock and labor. In eq. (3.2), the production technology is concave and strictly increasing in capital and labor, and shows the normal monotonicity: $\partial Y/\partial K_{ij} > 0 > \partial^2 Y/\partial K_{ij}^2$, $\partial Y/\partial L_{ij} > 0 > \partial^2 Y/\partial L_{ij}^2$, and $\partial^2 Y/\partial L_{ij}\partial K_{ij} > 0$. Also, it satisfies the Inada conditions for the two arguments: $\lim_{K_{ij}\to 0} \partial Y/\partial K_{ij} = \lim_{L_{ij}\to 0} \partial Y/\partial L_{ij} = \infty$, and $\lim_{K_{ij}\to\infty} \partial Y/\partial K_{ij} = \lim_{L_{ij}\to\infty} \partial Y/\partial L_{ij} = 0$.

In addition to the two production factors, the production function includes the aggregate level of capital stock K_j which implies a technological progress of jurisdiction j at each point in time t. The technological advance is considered as by-products when capitalists invest their stock of capital in the jurisdiction. Through the technological diffusion as a positive externality of capital stock, the local workers devote more effective labor to the local production and, in turn, get higher wages as in Arrow (1962).[8] Although it generates a negative environmental externality, the aggregate capital stocks K_j induced in the local production creates a positive externality on local workers' productivity. Like a pure public good within its boundary, the positive externality of one jurisdiction does not spill over to another. Hence, the level of labor efficiency differs across jurisdictions if the total amount of capital stock in the federation is distributed unevenly to each local jurisdiction over time.

[8]This type of technological advance is 'labor-augmenting' as in learning-by-doing models.

3.2.3 Firms

Given a rental price of capital r_j and a wage rate w_j in the jurisdiction j, each competitive firm i has a profit flow $\pi_{ij} = L_{ij}(F(k_{ij}, K_j) - r_j k_{ij} - w_j)$ at each point in time, where the function $F(k_{ij}, K_j) = Ak_{ij}^{\alpha} K_j^{1-\alpha}$ shows constant returns to scale in the capital-labor ratio $k_{ij} := K_{ij}/L_{ij}$ and the aggregate capital stock K_j. A single firm is so small that its own contribution to the aggregate capital stock of the jurisdiction is negligible. Hence, taken the technological progress in the jurisdiction K_j as parametric and using the zero-profit condition, the firms' profit maximization gives the rental price of capital and the wage rate where $r_j = \partial F(k_{ij}, K_j)/\partial k_{ij}$ and $w_j = F(k_{ij}, K_j) - k_{ij}\partial F(k_{ij}, K_j)/\partial k_{ij}$. Hence, the capitalists who invest their stock of capital into jurisdiction j earn a rate of return r_j equal to the marginal product of capital, and the workers who reside in the jurisdiction receive a wage rate w_j equal to marginal product of labor at each time t.

In each jurisdiction j, the aggregate level of capital stock and labor supply are $K_j = \int_0^1 K_{ij}\,\mathrm{d}i$ and $L_j = \int_0^1 L_{ij}\,\mathrm{d}i$, respectively. Assume that the workers in a jurisdiction supply their labors inelastically and normalize the aggregate labor supply L_j to one. Since in equilibrium, all firms in a jurisdiction choose the same levels of capital and labor, $L_{ij} = L_j = 1$ and $k_{ij} = K_j$ for each i, the equilibrium rental price of capital and the equilibrium wage rate of jurisdiction j is as follows:[9]

$$r_j = \alpha A \tag{3.3}$$

$$w_j = (1-\alpha)AK_j \qquad \text{for any } j. \tag{3.4}$$

The wage rate received by the local workers in jurisdiction j is increasing in the amount of capital stock attracted in the jurisdiction, whereas the rental price of capital is constant over time. Thus, in order to raise the wage rate of its workers as actual residents, each local government competes against the rest to induce the scarce capital stocks of the federation by using its policy variables at each time.

3.2.4 Capital mobility

Taking a rental price r_j and a tax rate of capital τ_j, in jurisdiction j as given, a representative capitalist who plans to invest his stock of capital in jurisdiction j chooses a time path of consumption $\{C_{cj}\}_{t=0}^{\infty}$ to maximize his discounted life-time utility

$$\int_0^\infty e^{-\rho t}\,\frac{C_{cj}^{1-\varepsilon}-1}{1-\varepsilon}\,\mathrm{d}t \tag{3.5}$$

subject to his flow budget constraint

$$C_{cj} + \dot{K}_j = (r_j - \tau_j)K_j, \tag{3.6}$$

[9]In equilibrium, $L_j = \int_0^1 L_{ij}\,di = L_{ij} = 1$ and in turn, $K_j = \int_0^1 K_{ij}\,di = K_{ij} = L_{ij}k_{ij} = k_{ij}$ for all t.

where a dot above a variable indicates the time derivative. The two parameters ε and ρ stand for a constant relative risk aversion (or inverse of elasticity of intertemporal substitution) and a rate of time preference of the capitalist respectively. In eq. (3.5), the instantaneous utility function of capitalist does not include the pollution generated in jurisdiction j (or the environmental quality of the jurisdiction). Because a regional environment is modeled as pure public good that can be consumed only within a particular region, the capitalist whose domicile is not the jurisdiction j need not be concerned about environmental quality of the jurisdiction when he invests his capital stock in the region. Even though his home is in the jurisdiction j as his capital location, the capitalist does not have to care about the environmental quality. Since he is able to separate his own stock of capital physically and spatially, the capitalist can leave the jurisdiction j with no relocation of the capital stocks.[10]

Inserting the rental price of capital of jurisdiction j in eq. (3.3), the maximization problem of the representative capitalist yields the capital accumulation equation in jurisdiction j in equilibrium

$$\dot{K}_j = (1/\varepsilon)(\alpha A - \tau_j - \rho)K_j, \tag{3.7}$$

when the capital stock is perfectly immobile.[11] However, in the context of interjurisdictional competition, the accumulation equation has to be modeled

[10] Actually, the model assumption of 'infinitesimal' local jurisdictions gives zero probability on the event that a domicile and a investment location of a capitalist is identical.

[11] The Hamiltonian of the representative capitalist reads: $\mathcal{H}_c(C_{cj}, K_j, \nu_j) = (C_{cj}^{1-\varepsilon} - 1)/(1-\varepsilon) + \nu_j((r_j - \tau_j)K_j - C_{cj})$. Differentiating this yields the first-order conditions with

as a mobility of capital stock. As in Rauscher (2005), the capital mobility term is augmented in eq. (3.7) as follows:

$$\dot{K}_j = (1/\varepsilon)(\alpha A - \tau_j - \rho)K_j + \phi(\alpha A - \tau_j - r_f)\psi(K_j, K_f), \qquad (3.8)$$

where $K_f = \int_0^1 K_j \mathrm{d}j$ is the total amount of capital stock, and r_f is denoted as a rate of return in the federation for each point in time.[12] In eq. (3.8), the second term shows a parameter $\phi \in [0, \infty)$ that measures a degree of capital mobility. If ϕ is zero, then capital stock is immobile across jurisdictions, and thus, eq. (3.8) reduces to eq. (3.7). The larger the parameter ϕ is, the more increased the capital mobility is. If ϕ tends to infinity, capital stock gets perfect mobility across jurisdictions. The size of investment flow is represented as a function $\psi(K_j, K_f)$ which is increasing in the capital stocks K_j in the jurisdiction itself and total capital stocks K_f in the federation. Further, the function ψ is assumed to be homogenous of degree one such that $\psi(K, K) = K$. Thus, a jurisdiction j can induce outside capital stocks into its region by cutting its capital tax rate, since the net rate of return in the jurisdiction is greater than

respect to the consumption C_{cj}, the capital stock K_j: $C_{cj}^{-\varepsilon} = \nu_j$ and $r_j - \tau_j = \rho - \dot{\nu}_j/\nu_j$. The first two conditions with eqs. (3.3) and (3.4) give the consumption growth rate of capitalist: $\dot{C}_{cj}/C_{cj} = (\alpha A - \tau_j - \rho)/\varepsilon$. For a growth path to be balanced, the net rate of return of capital, $\alpha A - \tau_j$, the capital tax rate τ_j should be constant over time. Thus, using the budget constraint in eq. (3.6) together with the consumption growth rate, it is shown that the capital and consumption grow at a same rate on the balanced growth path, and we arrive at eq. (3.7).

[12]As mentioned by Rauscher (2005), this formulation for capital mobility is intuitive and reasonable, whereas it is not derived explicitly by capitalists' profit-maximizing behavior. But, the formulation is easily applicable for analyzing the impact of the increased capital mobility. Of course, there are other specifications as in Lejour and Verbon (1997) and Hayashi (1982). But, the specifications seem to be not tractable or too complicated for the analysis. See Rauscher (2005) for more discussion and details.

in the rest ($\alpha A - \tau_j > r_f$).[13] Furthermore, the lower capital tax rate also raises the investment within the jurisdiction as seen in the first term in eq. (3.8). Since an increase in capital stock yields a higher wage rate for workers in eq. (3.4), local jurisdictions have incentives to reduce their capital tax rate to attract the capital stocks of the federation.

3.2.5 Local residents

At each point in time, the infinitely-lived workers have utility from consumption C_{wj} but disutility from polluting emission P_j in a jurisdiction j as taken the life-time utility function

$$W = \int_0^\infty e^{-\delta t}\, U(C_{wj}, P_j)\, \mathrm{d}t \tag{3.9}$$

and the instantaneous utility function

$$U(C_{wj}, P_j) = \begin{cases} \dfrac{(C_{wj} P_j^{-\eta})^{1-\sigma} - 1}{1-\sigma} & \text{for } 0 < \sigma < 1 \text{ and } \sigma > 1, \\ \ln C_{wj} - \eta \ln P_j & \text{for } \sigma = 1. \end{cases} \tag{3.10}$$

In eq. (3.9), the parameter δ denotes the rate of time preference of worker. In eq. (3.10), two parameters σ and η represents the inverse of intertemporal substitution and the weight for pollution respectively.[14] Generally, the rate of time preference and the inverse of intertemporal substitution for the workers need not be the same as for the capitalists (i.e. $\delta \neq \rho$ and $\sigma \neq \varepsilon$). Eq.

[13]However, the after-tax rental price of capital in the jurisdiction j is equal to the rate of return in the rest of federation in equilibrium, i.e. $\alpha A - \tau_j = r_f$.

[14]For a balanced growth path to be optimal, an instantaneous utility function is required to be an isoelastic form.

(3.10) says that the instantaneous function is strictly concave and increasing in private consumptions: $\partial U/\partial C_{wj} > 0 > \partial^2 U/\partial C_{wj}^2$. But, it is decreasing in the pollution generated in the jurisdiction j: $\partial U/\partial P < 0$. Furthermore, it satisfies the Inada conditions for consumption and environmental quality: $\lim_{C_{wj}\to 0} U_C = \lim_{P_j\to\infty} U_P = \infty$, and $\lim_{C_{wj}\to\infty} U_C = \lim_{P_j\to 0} U_P = 0$.[15]

To preserve a stable level of environmental quality, the authority of jurisdiction j finances the public abatement activity G_j with the tax revenue $\tau_j K_j$ collected from capital stock induced in the jurisdiction. Then the net of tax revenue (or a tax if it is negative), $T_j = \tau_j K_j - G_j$, is distributed equally to all workers in the jurisdiction. Thus, each worker's income consists of a wage rate w_j in eq. (3.4) and the tax revenues T_j at each point in time t. The flow budget constraint for a representative worker is

$$\begin{aligned} C_{wj} &= w_j + T_j \\ &= (1-\alpha)AK_j + (\tau_j K_j - G_j). \end{aligned} \tag{3.11}$$

Since the entry of more capital increases the wage rate of worker, each local jurisdiction has an incentive to reduce its capital tax rate against the rest of the federation to attract more mobile stock of capital. But, this lower capital tax rate can cause the jurisdiction to provide a lower level of public good for

[15]In this model, the inverse of a pollution level is equivalent to a level of environmental quality. Thus, zero pollution level that implies no production in the jurisdiction is equivalent to the infinite level of environmental quality. Substitute the pollution function (3.1) into the instantaneous utility function (3.10) to rewrite the following utility function of consumption and environmental quality: $U^*(C, G/K) = (C^{1-\sigma}(G/K)^{\chi\eta(1-\sigma)} - 1)/(1-\sigma)$. Then the Inada condition for environmental quality is $\lim_{G/K\to 0} U^*_{G/K} = \infty$ and $\lim_{G/K\to\infty} U^*_{G/K} = 0$. This requires the condition: $\chi\eta(1-\sigma) - 1 < 0$.

pollution abatement, and thus, the local environment of the jurisdiction can be more degraded.

3.3 Local outcomes with a full range of policies

This section examines the local setting with a full set of policy variables under no federal intervention. In other words, each single jurisdiction determines, as its own policy instruments, a capital tax rate and a level of local environmental quality.

3.3.1 Political mechanism

In order to investigate how an increased capital mobility impacts the local setting of policies instruments, and in turn, on the growth performance, the preservation of local environment, and welfare implication, the paper adopts a majority-voting model as in Oates and Schwab (1988). It is assumed further that capitalists who live in a particular jurisdiction vote with their feet as in Tiebout (1956) and Rauscher (2005). The capitalists can reflect their preferences and interests on local political issues by leaving the jurisdiction, because they are perfectly mobile in the sense that their own capital stocks can be physically and spatially separated from themselves. The capitalists need not choose a jurisdiction both for a home and an investment place. No matter where they reside, they can relocate the capital stocks in any regions. Thus, the capitalists are assumed not to participate in any local political issues in this model. However, workers have to live where they work. The labor as a factor of production is not physically and spatially divisible from the workers. Consequently, the workers become actual residents in a particular jurisdiction by participating in the local political procedure, and thus, the local government reflects only workers' interests.[16]

3.3.2 Sustainable development

Since all workers are homogenous in a jurisdiction, the outcome of a median voter is that of the maximization problem of the representative worker. In each jurisdiction j, a representative worker chooses a time path of capital tax rate, and public abatement good to maximize the discounted life-time welfare (3.9) subject to the flow budget constraint (3.11), and the accumulation equation of capital stock invested into the jurisdiction (3.8), given the initial amount of capital stock $K(0)$. The current-value Hamiltonian for the maximization problem of the representative worker reads

$$\begin{aligned} \mathcal{H}_w(\tau_j, G_j, K_j, \mu_j) &= U((1-\alpha)AK_j + (\tau_j K_j - G_j), P_j) \\ &\quad + \mu_j((1/\varepsilon)(\alpha A - \tau_j - \rho)K_j + \phi(\alpha A - \tau_j - r_j)\psi(K_j, K_f)), \end{aligned} \tag{3.12}$$

where μ_j denote the costate variable associated with the accumulation equation of capital stock invested in jurisdiction j (or the shadow price of capital stock).

[16]On the static model with interjurisdictional competition for a limited amount of capital stock in a federation, Oates and Schwab (1988) consider workers with no capital stocks as the residents of a particular jurisdiction.

The ex-post equilibrium conditions are given by

$$r_f = \alpha A - \tau, \tag{3.13}$$

$$\psi(K, K_f) = K, \tag{3.14}$$

since all identical jurisdictions choose the same levels of endogenous variables: $\tau_j = \tau$, $G_j = G$, $K_j = K$, $C_{wj} = C_w$, and $\mu_j = \mu$ for all j.[17] Then the current-value Hamiltonian (3.12) and eqs. (3.13) and (3.14) require the following first-order conditions with respect to the capital tax τ, the public abatement good G, the capital stock K, and the costate variable μ in equilibrium:

$$U_C = \mu(1/\varepsilon + \phi), \tag{3.15}$$

$$U_C = -\chi \frac{P}{G} U_P, \tag{3.16}$$

$$((1-\alpha)A + \tau)U_C - \chi \frac{P}{K} U_P + \mu(\alpha A - \tau - \rho)/\varepsilon = \delta\mu - \dot{\mu}, \tag{3.17}$$

$$\dot{K} = (1/\varepsilon)(\alpha A - \tau - \rho)K. \tag{3.18}$$

An additional condition that the optimal paths have to satisfy is the transversality condition

$$\lim_{t\to\infty} e^{-\delta t}\, \mu(t)K(t) = 0, \tag{3.19}$$

which guarantees that the consumption C_w and the capital stock K remain bounded at infinity.[18]

[17] In equilibrium, $K_f = \int_0^1 K_j dj = K \int_0^1 dj = K$ since $K_j = K$ for all j. Thus, the federal level of capital stock of the federation is the same as that of any jurisdiction: $K_f = K = K_j$ for all j. This fact together with the assumption that ψ is homogenous of degree implies that $\psi(K_j, K_f) = \psi(K, K) = K$.

[18] This condition is necessary for the representative worker's problem if her instantaneous utility is bounded.

Eq. (3.16) can be rewritten as

$$\frac{G}{K} = \chi\eta\frac{C_w}{K}. \tag{3.20}$$

Eq. (3.20) shows that under a full set of policy instruments, each local jurisdiction efficiently provides its environmental quality as local public good since the marginal utility of consumption is equal to the marginal utility of abatement.

Taking log and differentiating eq. (3.15) with respect to time and substituting this result together with eqs. (3.15) and (3.16) into eq. (3.17), we get the Keynes-Ramsey rule

$$\begin{aligned}\frac{\dot{C}_w}{C_w} = &\left[\left(A - \frac{G}{K} - \rho\right)/\varepsilon + \phi\left((1-\alpha)A + \tau - \frac{G}{K}\right) - \delta\right]/\sigma \\ &+ \eta(1 - 1/\sigma)\frac{\dot{P}}{P}\end{aligned} \tag{3.21}$$

which describes the optimal consumption of the workers in each jurisdiction over time for a given capital mobility. The last term of eq. (3.21) disappears, if the level of pollution is constant over time (that is, the capital stock K and public abatement activity G grow at a same rate) or an intertemporal elasticity of substitution $1/\sigma$ equals unity). The term $(A - G/K - \rho)/\varepsilon + \phi((1-\alpha)A + \tau - G/K)$ represents the rate of return to capital stock induced to each jurisdiction with respect to the point of worker's view. The rate of return to capital is divided by two terms. The first term $(A - G/K - \rho)/\varepsilon$ is the rate of return at zero capital mobility while the second term $\phi((1-\alpha)A + \tau - G/K)$ is the rate of return at a positive capital mobility which implies an externality of mobile stock of capital across jurisdictions. Given a constant level of pollution, the consumption growth rate of local worker is positive, zero, or negative if the rate of return to capital is larger than, equal to, or smaller than the rate of time preference for the worker. In addition, all other things being unchanged, the worker's consumption grows faster for an elasticity of intertemporal substitution smaller than one if the growth rate of pollution is larger. In the opposite case, the consumption grows slower for either of an intertemporal elasticity larger than unity or a negative growth rate of pollution.

Next, it is shown that all variables grow at a constant rate γ along a balanced growth path. By taking logs and differentiating eq. (3.20) with respect to time, the consumption C_w and the public abatement activity G grow at the same rate. For the growth path to be balanced, the the rate of return to capital in the federation is constant over time, and in turn, the capital tax rate τ is constant over time. Then substitute the ratio of consumption relative to capital from eq. (3.20) into the flow budget constraint of worker (3.11), and take logs and differentiate with respect to time to imply that the capital K and the abatement activity G grow at the same rate. Consequently, the transfer of net tax revenues T and the capital K grow at the same rate. Since the ratio of abatement relative to capital are constant, pollution P is constant as well. Thus, along a balanced growth path equilibrium, the stock of capital K, the consumption C_w of workers, and the abatement activities G of local government grow at the positive constant rate γ, but the capital tax rate τ

and the pollution amount P grow at a zero rate:[19]

$$\dot{\tau} = \dot{P} = 0, \tag{3.22}$$

$$\gamma := \frac{\dot{K}}{K} = \frac{\dot{C}_w}{C_w} = \frac{\dot{G}}{G} = \frac{\dot{T}}{T} = (\alpha A - \tau - \rho)/\varepsilon. \tag{3.23}$$

Proposition 3.3.1 (sustainable growth)**.** *Given a capital mobility $\phi \in [0, \infty)$, each local jurisdiction achieves sustainable growth with a stable level of local environmental quality. That is, the growth rate of workers' consumption is positive, while the growth rate of regional pollution is zero over time.*

3.3.3 The impact of increased capital mobility

To characterize the outcome of local setting of policy instruments and analyze the impact of increased capital mobility in a simplified framework, denote the abatement and the consumption relative to capital as fundamental variables that are constant on the balanced growth path as $g := G/K$ and $c := C/K$. In addition,

Definition 3.3.1. Let a function $\Delta_\phi : \mathbb{R}_+ \to (0, 1]$ be defined as

$$\Delta_\phi(x) = \frac{x}{\varepsilon\phi + x},$$

[19]It is obvious that modeling pollution as a flow variable gives a simplified framework, but it does not modify the qualitative results along a balanced growth path. If the local environment of each jurisdiction were modeled as a renewable resource as in Bovenberg and Smulders (1995), pollution could not grow over time, since it should not exceed the regeneration capacity of natural environment for a balanced growth path to be sustainable. This is in accordance with the formulation of this model. In order to stabilize pollution, an endogenously growing jurisdiction can provide an increasing number of public abatement activities.

which is decreasing in ϕ, and has the upper bound of unity, $\Delta_0(x) = 1$, at the zero mobility and the lower bound of zero, $\Delta_\infty(x) = \lim_{\phi\to\infty} \Delta_\phi(x) = 0$, at the perfect mobility respectively for any positive value $x \in \mathbb{R}_+ := (0, \infty)$.

Then eqs. (3.11), (3.20), (3.21), (3.22), and (3.23) give the reduced forms of solutions as follows:[20]

$$\gamma = \frac{A - \rho - (1+\chi\eta)\bar{c}_w \Delta_\phi(\kappa)}{\varepsilon}, \quad (\textit{growth rate}) \tag{3.24}$$

$$\tau = -(1-\alpha)A + (1+\chi\eta)\bar{c}_w \Delta_\phi(\kappa), \quad (\textit{capital tax rate}) \tag{3.25}$$

$$g = \chi\eta\bar{c}_w \Delta_\phi(\kappa), \quad (\textit{public abatement}) \tag{3.26}$$

$$c_w = \bar{c}_w \Delta_\phi(\kappa), \quad (\textit{consumption}) \tag{3.27}$$

where the parameter κ and the consumption-capital ratio $\bar{c}_w$ at zero mobility ($\phi = 0$) are denoted as

$$\bar{c}_w := \frac{\varepsilon\delta - (1-\sigma)(A-\rho)}{\kappa} \quad \text{and} \quad \kappa := 1 - (1+\chi\eta)(1-\sigma).$$

The following lemma states that this model requires two inequalities for positive growth rate and bounded utility respectively.

[20]The original solutions for the growth rate γ, the capital tax rate τ, and the ratio of public abatement g and the consumption c_w relative to capital have the following reduced forms:

$$\begin{cases} \gamma = \left(A - \rho - \frac{(1+\chi\eta)(\delta-(1-\sigma)(A-\rho)/\varepsilon)}{\phi+(1-(1+\chi\eta)(1-\sigma))/\varepsilon}\right)/\varepsilon, \\ \tau = -(1-\alpha)A + \frac{(1+\chi\eta)(\delta-(1-\sigma)(A-\rho)/\varepsilon)}{\phi+(1-(1+\chi\eta)(1-\sigma))/\varepsilon}, \\ g = \frac{\chi\eta(\delta-(1-\sigma)(A-\rho)/\varepsilon)}{\phi+(1-(1+\chi\eta)(1-\sigma))/\varepsilon}, \\ c_w = \frac{\delta-(1-\sigma)(A-\rho)/\varepsilon}{\phi+(1-(1+\chi\eta)(1-\sigma))/\varepsilon} \end{cases}$$

from eqs. (3.11), (3.20), (3.21), (3.22), and (3.23).

Lemma 3.3.2. *Suppose this endogenous growth model satisfy the following two inequalities:*

$$A > \varepsilon\delta(1+\chi\eta) + \rho \quad \textit{and} \quad \sigma > 1 - \frac{\varepsilon\delta}{A-\rho}. \tag{3.28}$$

Then (i) $A-\rho > 0$ *and* $\kappa > 0$*, and in turn,* (ii) *the sustainability and transversality conditions are satisfied.*

Proof. See Appendix A. □

The next thing we do is to investigate the impact of an increase in capital mobility on regional growth and environment. Differentiating the growth rate γ in eq. (3.24) and the abatement-capital ratio g in eq. (3.26) with respect to the parameter ϕ of capital mobility yields

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\phi} = -\frac{(1+\chi\eta)\bar{c}_w}{\varepsilon} \cdot \frac{\mathrm{d}\Delta_\phi(\kappa)}{\mathrm{d}\phi} > 0,$$
$$\frac{\mathrm{d}g}{\mathrm{d}\phi} = \chi\eta\bar{c}_w \cdot \frac{\mathrm{d}\Delta_\phi(\kappa)}{\mathrm{d}\phi} < 0,$$

since Δ is decreasing in ϕ by Definition 1. Thus, the growth rate γ is positively related to ϕ while the abatement-capital ratio g is negatively related to ϕ. Moreover, the capital tax rate τ becomes negative in eq. (3.25) and the abatement-capital ratio g is zero in eq. (3.26), as the stock of capital is perfectly mobile ($\phi \to \infty$). That is,

$$\begin{aligned}\lim_{\phi\to\infty} \tau &= \lim_{\phi\to\infty} [-(1-\alpha)A + (1+\chi\eta)\bar{c}_w\Delta_\phi(\kappa)] \\ &= -(1-\alpha)A + (1+\chi\eta)\bar{c}_w\Delta_\infty(\kappa) = -(1-\alpha)A < 0,\end{aligned}$$

$$\lim_{\phi\to\infty} g = \lim_{\phi\to\infty} \chi\eta\bar{c}_w\Delta(\phi,\kappa) = \chi\eta\bar{c}_w\Delta_\infty(\kappa) = 0.$$

The increased capital mobility gradually reduces the capital tax rate. Note that each local economy has an infinite amount of pollution at $g = 0$. Even if it positively affects the growth of local workers' consumption, the capital mobility gives a negative effect on local environment. Hence the increase in capital mobility generates a trade-off between a higher growth rate and a lower environmental quality to local jurisdictions.

To investigate the effect of increased capital mobility on the welfare of residents, the life-time welfare function (3.9) is integrated with respect to time as follows:

$$W = \begin{cases} \dfrac{1}{1-\sigma}\left[\dfrac{K(0)^{1-\sigma}(c_w)^{1-\sigma}P^{-\eta(1-\sigma)}}{\delta-\gamma(1-\sigma)} - \dfrac{1}{\delta}\right] & \text{for } 0<\sigma<1 \text{ and } \sigma>1, \\ \dfrac{1}{\delta}K(0) + \dfrac{1}{\delta}\ln c_w - \dfrac{1}{\delta}\eta\ln P + \dfrac{1}{\delta^2}\gamma & \text{for } \sigma = 1. \end{cases} \tag{3.29}$$

Plugging the growth rate (3.24), the abatement-capital ratio (3.26), and the consumption-capital ratio (3.27) into eq. (3.29) gives the present value of the workers' welfare as

$$W = \frac{1}{1-\sigma}\left[((1-\sigma)\bar{W} + \frac{1}{\delta})\Delta_\phi(1)\Delta_\phi(\kappa)^{-\kappa} - \frac{1}{\delta}\right] \quad (\textit{welfare}) \tag{3.30}$$

where $(1-\sigma)\bar{W} + 1/\delta = \varepsilon K(0)^{1-\sigma}(\bar{c}_w)^{-\sigma+\chi\eta(1-\sigma)}\chi\eta^{\chi\eta(1-\sigma)} > 0$. Differentiating eq. (3.30) gives

$$\frac{\mathrm{d}W}{\mathrm{d}\phi} = -\frac{((1-\sigma)\bar{W} + 1/\delta)(1+\chi\eta)\varepsilon^2}{\kappa}\cdot\phi\Delta_\phi(1)^2\Delta_\phi(\kappa)^{1-\kappa} < 0.$$

The life-time welfare of workers is decreasing in capital mobility. The results are summarized formally in

Proposition 3.3.3 (full set of local policies)**.** *Suppose that the capital mobility ϕ is increased. Then the growth rate γ increases, but the abatement-capital ratio g decreases in each local jurisdiction. Moreover, the capital tax rate τ becomes negative, and the deterioration of the environment is complete with the perfect mobility of capital. The increased capital mobility reduces the residents' welfare W.*

"Tax importing" dominates the growth effect as the capital stock becomes more mobile. Even though it provides local jurisdictions with a higher growth rate, the increased capital mobility transfers the burden of pollution abatement from capitalists to workers in each jurisdiction. The higher capital mobility prevents the jurisdiction from taxing on capital, and it reinforces their "race to the bottom" in the environmental standards.

3.4 Federal interventions

If the federation does not intervene in the local setting, an increase in capital mobility gradually degrades local environments and reduces the residents' welfare. Hence, the next two subsections explore alternative federal policies that keep the residents' welfare at a higher level against the increased mobility of capital.

3.4.1 Uniform environmental standard

The competing jurisdictions with no federal mediation suffer from lower environmental quality as the stock of capital gets more mobile. To save the regional environments from deterioration, a federation has an incentive to set a uniform environmental standard for the local jurisdictions. Hence this subsection examines how a higher level of government sets the uniform standard for lower levels of government. At the beginning of time, the federal government considers a particular amount of pollution for local environments such that $P_f = P_j$ for all j and any t. Then the uniform environmental standard can be defined by a public abatement activity G_j relative to capital K_j induced into each jurisdiction j:

$$\zeta := \frac{G_j}{K_j} = \left(\frac{1}{P_f}\right)^{1/\chi}. \tag{3.31}$$

Suppose the federal government gives an environmental guideline ζ to the local governments. Then each jurisdiction sets a capital tax rate as its own policy instrument. Thus backward induction characterizes an optimal federal standard and local equilibrium outcomes. Regarding the uniform standard as given, the representative worker first sets a capital tax rate in each jurisdiction. Then, subject to this local outcomes, the federal authority chooses a level of environmental standard to maximize local workers' welfare. That is, this two-stage problem endogenously determines the uniform environmental standard in the model.

Now, a representative worker as a median voter in each jurisdiction j chooses a time path of capital tax rate to optimize his life-time utility (3.9), subject to the flow budget constraint (3.11), the accumulation of mobile capital (3.8), and the federal uniform environmental standard (3.31), given an amount of capital stock $K(0)$ in his jurisdiction at the beginning of the time.

Therefore, this maximization problem reads the current-value Hamiltonian:

$$\begin{aligned}\mathcal{H}_w(\tau_j, K_j, \mu_j) &= U(((1-\alpha)A + \tau_j - \zeta)K_j, \zeta^{-\chi}) \\ &\quad + \mu_j((1/\varepsilon)(\alpha A - \tau_j - \rho)K_j + \phi(\alpha A - \tau_j - r_f)\psi(K_j, K_f)). \quad (3.32)\end{aligned}$$

Appendix B.1 derives as local outcomes the growth rate, the capital tax rate, and the consumption-capital ratio under a given abatement-capital ratio ζ. Thus,

$$\gamma(\zeta) = \left[A - \rho - \frac{\varepsilon\delta - (1-\sigma)(A-\rho) + \zeta(\varepsilon\phi + 1)}{\varepsilon\phi + \sigma}\right]/\varepsilon, \quad (3.33)$$

$$\tau(\zeta) = -(1-\alpha)A + \frac{\varepsilon\delta - (1-\sigma)(A-\rho) + \zeta(\varepsilon\phi + 1)}{\varepsilon\phi + \sigma}, \quad (3.34)$$

$$c_w(\zeta) = \frac{\varepsilon\delta - (1-\sigma)(A-\rho-\zeta)}{\varepsilon\phi + \sigma} > 0. \quad (3.35)$$

The federal authority then chooses an abatement-capital ratio ζ to maximize the integrated life-time utility (3.29) subject to local outcomes in eqs. (3.33), (3.34), and (3.35). Appendix B.1 derives the first-order condition with respect to ζ as

$$\frac{\mathrm{d}W}{\mathrm{d}\zeta} = \frac{(W(1-\sigma) + 1/\delta)\kappa}{\varepsilon\delta - (1-\sigma)(A-\rho-\zeta)} \cdot \left(\frac{\chi\eta\bar{c}_w}{\zeta} - 1\right) = 0, \quad (3.36)$$

where $W(1-\sigma)+1/\delta = K(0)^{1-\sigma}c_w^{1-\sigma}P^{-\eta(1-\sigma)}/(\delta-\gamma(1-\sigma))$ is a positive term. Since the left multiplier is positive in eq. (3.36), the optimal uniform standard ζ^* of environmental quality is given as

$$\zeta^* = \chi\eta\bar{c}_w. \quad (\textit{public abatement}). \quad (3.37)$$

which is not related to a positive capital mobility ϕ. Plugging eq. (3.37) into eqs. (3.33), (3.34), and (3.35) gives the equilibrium local outcomes

$$\gamma(\zeta^*) = \frac{A - \rho - \bar{c}_w(\Delta_\phi(\sigma) + \chi\eta)}{\varepsilon}, \quad (\textit{growth rate}) \tag{3.38}$$

$$\tau(\zeta^*) = -(1-\alpha)A + \bar{c}_w(\Delta_\phi(\sigma) + \chi\eta), \quad (\textit{capital tax rate}) \tag{3.39}$$

$$c_w(\zeta^*) = \bar{c}_w \Delta_\phi(\sigma), \quad (\textit{consumption}) \tag{3.40}$$

where the superscript * indicates that the uniform standard is at the optimal level. To examine how this optimal uniform standard changes the residents' welfare, the substitution of eqs. (3.37), (3.38), and (3.40) into eq. (3.29) evaluates the integrated utility as

$$W(\zeta^*) = \frac{1}{1-\sigma}\left[\left((1-\sigma)\bar{W} + \frac{1}{\delta}\right)\Delta_\phi(1)\Delta_\phi(\sigma)^{-\sigma} - \frac{1}{\delta}\right] \quad (\textit{welfare}) \tag{3.41}$$

at the optimal level of environmental standard. Thus,

Proposition 3.4.1 (uniform environmental standard)**.** *Suppose that the mobility of capital stock is positive, $\phi \in (0, \infty)$. Then each local jurisdiction has* (i) *a lower growth rate $\gamma(\zeta^*)$ and* (ii) *a higher abatement-capital ratio ζ^* that is independent of ϕ under the optimal uniform environmental standard ζ^* than under no federal intervention. The federal uniform standard improves* (iii) *the local residents' welfare $W(\zeta^*)$ better for $\sigma \neq 1$ or equal for $\sigma = 1$. If the mobility of capital stock is zero, $\phi = 0$, then all the local outcomes and welfare level under the optimal uniform environmental standard are the same as under no federal intervention.*

Proof. See Appendix A. □

Eq. (3.37) states that the optimal uniform environmental standard is independent of any capital mobility $\phi \in [0, \infty)$. The optimal uniform standard preserves the regional environments at the same level as the stock of capital is perfectly immobile ($\phi = 0$). Given a level of capital mobility, the consumption growth rate is relatively lower in eq. (3.38). However, the welfare of local residents is improved under the uniform environmental standard, since the standard saves local environments from degradation by a higher mobility of capital stock (i.e. severe competition).

3.4.2 Requirement of lump sum transfer (or tax)

Section 3 observes a reduction in the consumption level of local residents relatively to the amount of capital invested in a jurisdiction, if the capital mobility increases. In this case, the federal government may imposes a requirement of lump sum transfer (or tax) in a redistributive objective. Then local jurisdictions should use the capital tax as a second best, to finance their public abatement activities. According to the local public finance literature, the local environment as a public good might be under-provided in this case.[21] This subsection investigates how the federal restriction on lump sum transfer (or tax) provides the regional environment in this context. The federation is assumed to require a fixed ratio of lump sum transfer to localities at the beginning of the time. This federal requirement can then be defined by the

[21] See this result in Oates and Schwab (1988).

lump sum transfer relative to capital:

$$\xi := \frac{T_j}{K_j}. \tag{3.42}$$

To derive the optimal requirement of lump sum transfer, backward induction is utilized again in the same way as in the previous subsection. Instead of eq. (3.11), the budget constraint of workers then becomes

$$C_{wj} = ((1-\alpha)A + \xi)K_j. \tag{3.43}$$

A representative worker (as a median voter) maximizes his life-time welfare (3.9) subject to the above budget constraint (3.43), and the accumulation of capital stock in eq. (3.8) by choosing a time path of capital tax rate, given $K(0)$ as an initial capital amount in the jurisdiction. The current-value Hamiltonian for this optimization is

$$\begin{aligned}\mathcal{H}_w(\tau_j, K_j, \mu_j) &= U(((1-\alpha)A + \xi)K_j, (\tau_j - \xi)^{-\chi}) \\ &\quad + \mu_j((1/\varepsilon)(\alpha A - \tau_j - \rho)K_j + \phi(\alpha A - \tau_j - r_f)\psi(K_j, K_f)). \quad (3.44)\end{aligned}$$

Appendix B.2 derives as regional outcomes the growth rate, capital tax rate, and abatement-capital ratio

$$\gamma(\xi) = \left[A - \rho - \frac{\chi\eta(\varepsilon\delta - (1-\sigma)(\alpha A - \rho)) - \xi(1+\varepsilon\phi)}{\varepsilon\phi + \iota}\right] / \varepsilon, \tag{3.45}$$

$$\tau(\xi) = \frac{\chi\eta(\varepsilon\delta - (1-\sigma)(\alpha A - \rho)) + \xi(1+\varepsilon\phi)}{\varepsilon\phi + \iota}, \tag{3.46}$$

$$g(\xi) = \frac{\chi\eta(\varepsilon\delta - (1-\sigma)(\alpha A - \rho - \xi))}{\varepsilon\phi + \iota} > 0, \tag{3.47}$$

which optimally adjust to any federal transfer requirement ξ. Then the federation chooses ξ to optimize the integrated life-time utility (3.29) subject to regional outcomes in eqs. (3.45), (3.46), and (3.47). Appendix B.2 derives the first-order condition with respect to the consumption-capital ratio ξ:

$$\frac{\mathrm{d}W}{\mathrm{d}\xi} = \frac{(W(1-\sigma)+1/\delta)\kappa}{\varepsilon\delta-(1-\sigma)(\alpha A-\rho-\xi)} \cdot \left(\frac{\bar{c}_w}{(1-\alpha)A+\xi}-1\right) = 0. \tag{3.48}$$

Note that the left multiplier is positive in eq. (3.48). Thus, the optimal federal requirement ξ^* of lump sum transfer is given as

$$\xi^* = -(1-\alpha)A + \bar{c}_w \tag{3.49}$$

which, in turn, leads to the consumption-capital ratio

$$c_w(\xi^*) = \bar{c}_w \quad (\textit{consumption}) \tag{3.50}$$

by the budget constraint in eq. (3.43). The optimal consumption-capital ratio $c_w(\xi^*)$ does not depend on any positive capital mobility ϕ. Substituting eq. (3.49) into the local outcomes in eqs. (3.45), (3.46), and (3.47) yields equilibrium local outcomes

$$\gamma(\xi^*) = \frac{A-\rho-\bar{c}_w(1+\chi\eta\Delta_\phi(\iota))}{\varepsilon}, \quad (\textit{growth rate}) \tag{3.51}$$

$$\tau(\xi^*) = -(1-\alpha)A + \bar{c}_w(1+\chi\eta\Delta_\phi(\iota)), \quad (\textit{capital tax rate}) \tag{3.52}$$

$$g(\xi^*) = \chi\eta\bar{c}_w\Delta_\phi(\iota), \quad (\textit{public abatement}) \tag{3.53}$$

where $\iota := 1-\chi\eta(1-\sigma)$. To investigate how the optimal federal requirement ξ^* alters the residents' welfare, the substitution of eqs. (3.50), (3.51), and (3.53) into eq. (3.29) evaluates the integrated utility as

$$W(\xi^*) = \frac{1}{1-\sigma}\left[\left((1-\sigma)\bar{W} + \frac{1}{\delta}\right)\Delta_\phi(1)\Delta_\phi(\iota)^{-\iota} - \frac{1}{\delta}\right] \quad (welfare) \qquad (3.54)$$

at the optimal level of federal requirement ξ^*. Hence,

Proposition 3.4.2 (requirement of lump sum transfer)**.** *Suppose that the mobility of capital stock is positive, $\phi \in (0, \infty)$. Then each local jurisdiction has* (i) *a lower growth rate $\gamma(\xi^*)$ and* (ii) *a higher, equal, or lower abatement-capital ratio $g(\xi^*)$ for $\sigma < 1$, $\sigma = 1$, or $\sigma > 1$ under the optimal requirement of lump sum transfer ξ^* than under no federal intervention. The federal requirement improves* (iii) *the local residents' welfare $W(\xi^*)$ better for $\sigma \neq 1$ or equal for $\sigma = 1$. If the mobility of capital stock is zero, $\phi = 0$, then all the local outcomes and welfare level under the optimal requirement of lump sum transfer are the same as under no federal intervention.*

Proof. See Appendix A. □

Eq. (3.50) implies that the consumption relative to stock of capital is independent of any capital mobility $\phi \in (0, \infty)$. The optimal requirement keeps the consumption-capital ratio in the same level as the capital stock is perfectly immobile ($\phi = 0$). The consumption growth rate is relatively lower in eq. (3.51), since the federal requirement of lump sum transfer (or tax) enforces the local jurisdictions to tax on mobile capital. The regional environment is more degraded for $\phi \in (0, \infty)$, if the elasticity of intertemporal substitution is less than one ($\sigma > 1$). In contrast to this prediction as in static models with interjurisdictional competition, we have the opposite result as well, if the elasticity of intertemporal substitution is greater than one ($\sigma < 1$). That is, the regional environment as public good is over-provided even with the federal restriction on lump sum transfer (or tax). The local residents' welfare is enhanced under the requirement of lump sum transfer (or tax), since the requirement achieves the redistributive object against the increasing capital mobility (i.e. severe competition).

To complete welfare comparison among three different policy systems, I examine welfare difference between under uniform standard of environmental quality and requirement of lump sum transfer (or tax).

Corollary 3.4.3. *Suppose that the mobility of capital stock is positive, $\phi \in (0, \infty)$. Then the optimal requirement of lump sum transfer ξ^* makes the residents' welfare $W(\xi^*)$ better for $\chi\eta < 1$, equal for $\chi\eta = 1$, or worse for $\chi\eta > 1$ than the optimal uniform environmental standard ζ^*. If the mobility of capital stock is zero, $\phi = 0$, then the welfare level under the optimal requirement of lump sum transfer is the same as under the optimal uniform standard of environmental quality.*

Proof. See Appendix A. □

Using Propositions 3.4.1 – 3.4.2 and Corollary 3.4.3, Table 3.1 summarizes welfare comparison among three alternative policy structures. If the elasticity of intertemporal substitution is one ($\sigma = 1$), the residents in each jurisdiction have the same level of welfare among all three policy systems, and the welfare level does not depend on any positive level of mobility ($W = W(\zeta^*) = W(\xi^*) = \bar{W}$). Thus, any federal intervention has no effect on the residents' welfare. If the elasticity of intertemporal substitution is not one ($\sigma \neq 1$), either one of two federal interventions enhances the welfare level for any positive level of capital mobility, however. If $\sigma \neq 1$ and $\chi\eta = 1$, both of the interventions are equally effective. If $\sigma \neq 1$ and $\chi\eta > 1$, the uniform environmental standards should be preferred to the requirement of lump sum transfer (or tax). This is the case where the residents' environmental concern is relatively higher or the regional environment is relatively more polluted by the capital stock induced in the jurisdiction. On the other case ($\sigma \neq 1$ and $\chi\eta < 1$), the lump sum transfer (or tax) requirement is more effective than the uniform standard. Since the increased capital mobility transfers the burden of pollution abatement to each jurisdiction, the federal intervention is necessary to save the jurisdictions.

Table 3.1: Welfare Comparison among Three Alternative Policy Structures

	$\chi\eta < 1$	$\chi\eta = 1$	$\chi\eta > 1$
$\sigma = 1$	$W = W(\zeta^*) = W(\xi^*)$	$W = W(\zeta^*) = W(\xi^*)$	$W = W(\zeta^*) = W(\xi^*)$
$\sigma \neq 1$	$W < W(\zeta^*) < W(\xi^*)$	$W < W(\zeta^*) = W(\xi^*)$	$W < W(\zeta^*) > W(\xi^*)$

[a] σ: the inverse of intertemporal substitution
η the weight for pollution
χ: the positive elasticity of pollution with respect to the capital-abatement ratio.

[b] W: welfare with full set of local policies
$W(\zeta^*)$: welfare under a uniform environmental standard
$W(\xi^*)$: welfare under a federal requirement of revenue transfer

3.5 Conclusion

The paper identifies that an increase in capital mobility provides local jurisdictions with a higher growth rate. Since the increasing mobility of capital strengthens the jurisdictions to set a lower capital tax rate, the stock of capital can rapidly accumulate in the jurisdictions. Although it has a positive effect on the growth rate, the increase in capital mobility degrades regional environments. This finding supports the hypothesis of "race to the bottom" in environmental standards. Thus, the capital mobility presents a trade-off between growth rate and environmental quality. If the stock of capital is relatively more mobile, each jurisdiction cannot avoid to collect relatively less revenue from capital stock. To finance the local public expenditure on pollution abatement, the jurisdiction relies relatively more on the lump sum tax (or relatively less on the lump sum transfer). That is, each jurisdiction "imports tax" within its regional boundary. The increasing capital mobility transfers the burden of public funds from capitalists to local residents. Therefore, both of the federal interventions are meaningful and could perform key roles in reality.

I do not consider the jurisdictions where the workers as actual residents are heterogenous as in Oates and Schwab (1988). The impact of increasing capital mobility would be different according to alternative outcome from majority voting. To investigate this, one can easily extend this model, however. A majority of non-wage workers could be an alternative against the increasing mobility of capital. This outcome might reduce the negative effects on regional environments and welfare, since the non-wage workers depend relatively less on the amount of capital induced in the jurisdiction. But, the impact of increasing capital mobility is somewhat negative in the majority of wage workers. The income source of wage workers is very related to the level of capital stock. Thus the wage workers would vote for even lower capital tax rates as the stock of capital gets more mobile.

Appendices

Appendix A

Appendix of Chapter 1

A.1 Expenditure function approach

From the maximization problem of the individual taxpayer, we can figure out demand functions for C_1 and C_2, supply function for L, and tax evaded E as the optimal choices. Since we derive the first order conditions in eqs. (1.4) - (1.7) in section 3 and reformulate the two budget constraints with the virtual income Z (regarded as exogenously given) in section 4, the optimal choice functions C_1, C_2, L and E can have as their arguments the net wage rate $(1-m)w$, audit rate p, penalty rate π, publicly supplied nonmarket good G, and transfer Z. Thus, the indirect expected utility function $\bar{U}^*$ can be constructed as $\bar{U}^* = \bar{U}^*\left(\left(1-m\right)w, p, \pi, G, Z\right) \equiv (1-p)U(C_1\left(\cdot\right), V\left(\cdot\right), G) + pU(C_2\left(\cdot\right), V\left(\cdot\right), G)$ where the dot represents the vector of the arguments mentioned above. By using the Envelope Theorem, we have the partial derivatives of $\bar{U}^*$ with respect to each of the parameters $(1-m)w$, p, π, and Z as follows:

$$\bar{U}^*_{(1-m)w} = \left(\lambda_1 + \lambda_2\right) L, \tag{A.1}$$

$$\bar{U}^*_p = U(C_2, V, G) - U(C_1, V, G) \leq 0, \tag{A.2}$$

$$\bar{U}^*_\pi = -\lambda_2 E, \tag{A.3}$$

$$\bar{U}^*_Z = \lambda_1 + \lambda_2. \tag{A.4}$$

In order to develop the relationship between the uncompensated and compensated elasticity of labor supply with respect to each of the net wage rate $(1-m)w$, the audit rate p and fine rate θ, we exploit the expenditure function approach. Now, the uncompensated and compensated labor supply are equal at the optimum: $L\left((1-m)w, p, \pi, G, Z\left((1-m)w, p, \pi, \bar{U}^*\right)\right) = L^c\left((1-m)w, p, \pi, G, \bar{U}^*\right)$. Partially differentiate this identity to get three Slutsky equations that are associated with the net wage rate $(1-m)w$, the probability rate p and the penalty rate π:

$$L_{(1-m)w} + L_Z Z_{(1-m)w} = L^c_{(1-m)w}, \tag{A.5}$$

$$L_p + L_Z Z_p = L^c_p, \tag{A.6}$$

$$L_\pi + L_Z Z_\pi = L^c_\pi. \tag{A.7}$$

Since Z can be found by inverting the indirect utility function $\bar{U}^*$, it is easy to derive the partial derivatives of the virtual income Z from eqs. (A.1) - (A.4) together with eq. (1.6) as follows:

$$Z_{(1-m)w} = -\frac{\bar{U}^*_{(1-m)w}}{\bar{U}^*_Z} = -L, \tag{A.8}$$

$$Z_p = -\frac{\bar{U}^*_p}{\bar{U}^*_Z} = \frac{E}{p}, \tag{A.9}$$

$$Z_\pi = -\frac{\bar{U}^*_\pi}{\bar{U}^*_Z} = \frac{E}{\theta}. \tag{A.10}$$

The first-order Taylor expansion is used in order to approximate the marginal utility function $U\left(C_1, V, G\right)$ to $U\left(C_2, V, G\right) + U_C\left(C_2, V, G\right)\left(C_1 - C_2\right)$ in eq. (A.9). Insert eqs. (A.8) - (A.10) into three Slutsky equations above, and then multiply the results by $(1-m)\,w/L$, P/L and θ/L respectively to get three relationship between the uncompensated and compensated elasticity of labor supply with respect to the net wage, audit, and fine rate:

$$\eta - (1-m)\,wL_Z = \eta^c, \tag{A.11}$$

$$\varepsilon_p + \frac{E}{L}L_Z = \varepsilon_p^c, \tag{A.12}$$

$$\varepsilon_\theta + \frac{E}{L}L_Z = \varepsilon_\theta^c. \tag{A.13}$$

In eqs. (A.11) - (A.13), the superscript c indicates "compensated," while no superscript implies "uncompensated." The parameters η, ε_p, and ε_θ denote the elasticity of labor supply with respect to the net wage, audit, and fine rate respectively. Furthermore, combining eqs. (A.11) - (A.13) together yields

$$L_Z = -\frac{\eta^c - \eta}{(1-m)\,w} = \left(\varepsilon_p^c - \varepsilon_p\right)\frac{L}{E} = \left(\varepsilon_\theta^c - \varepsilon_\theta\right)\frac{L}{E}$$

which gives

$$\varphi \equiv \frac{E}{wL} = -\frac{(1-m)\left(\varepsilon_p^c - \varepsilon_p\right)}{\eta^c - \eta} = -\frac{(1-m)\left(\varepsilon_\theta^c - \varepsilon_\theta\right)}{\eta^c - \eta} > 0.$$

Appendix B

Appendix of Chapter 3

B.1 Proofs

Proof of Lemma 3.3.2. (i). Assume that eq. (3.28) are satisfied. Then $A-\rho > \varepsilon(1+\chi\eta)\delta > 0$. The parameter κ is positive for $\sigma \geq 1$. If $\sigma < 1$, then

$$\kappa = 1 - (1+\chi\eta)(1-\sigma) = 1 - \frac{\varepsilon(1+\chi\eta)\delta}{A-\rho} \cdot \frac{(A-\rho)(1-\sigma)}{\varepsilon\delta} > 0,$$

since $0 < \varepsilon\delta(1+\chi\eta)/(A-\rho) < 1$ and $0 < (A-\rho)(1-\sigma)/\varepsilon\delta < 1$ from rearranging eq. (3.28). (ii). The growth rate is positive for arbitrary $\phi \in [0,\infty)$:

$$\gamma = \frac{A-\rho-(1+\chi\eta)\bar{c}_w\Delta_\phi(\kappa)}{\varepsilon} = \frac{\varepsilon(A-\rho)\phi + A-\rho-\varepsilon\delta(1+\chi\eta)}{\varepsilon(\varepsilon\phi+\kappa)} > 0,$$

since $A-\rho > 0$ and $A-\rho-\varepsilon\delta(1+\chi\eta) > 0$ from using (i) and rearranging the first inequality in eq. (3.28). The transversality condition (3.19) is satisfied for arbitrary $\phi \in [0,\infty)$ as follows:

$$\lim_{t\to\infty} e^{-\delta t}\mu K = \lim_{t\to\infty} e^{-(\delta-\gamma(1-\sigma))t}\mu(0)K(0) = \lim_{t\to\infty} e^{-(\delta-\gamma(1-\sigma))t} \cdot \frac{U_C(0)\varepsilon K(0)}{\varepsilon\phi+1} = 0,$$

since

$$\delta - \gamma(1-\sigma) = c_w(1/\varepsilon+\phi) = \frac{\bar{c}_w\Delta_\phi(\kappa)}{\varepsilon\Delta_\phi(1)} = \frac{\varepsilon\delta-(1-\sigma)(A-\rho)}{\varepsilon\kappa} \cdot \frac{\Delta_\phi(\kappa)}{\Delta_\phi(1)} > 0,$$

and the term $U_C(0)\varepsilon K(0)/(\varepsilon\phi+1)$ is positive and finite. □

Proof of Proposition 3.4.1. (i). The difference of growth rates between under no federal intervention and under the optimal uniform environmental standard is

$$\begin{aligned}\gamma - \gamma(\zeta^*) &= -\frac{\bar{c}_w[(1+\chi\eta)\Delta_\phi(\kappa) - (\Delta_\phi(\sigma)+\chi\eta)]}{\varepsilon} \\ &= \frac{\chi\eta\bar{c}_w}{\kappa\sigma} \cdot \frac{\phi\Delta_\phi(\kappa)\Delta_\phi(\sigma)}{\Delta_\phi(1)} \\ &> 0 \quad \text{if } \phi > 0 \text{ and} \\ &= 0 \quad \text{if } \phi = 0.\end{aligned}$$

(ii). The difference of abatement-capital ratios between under no federal intervention and under the optimal uniform environmental standard is

$$\begin{aligned}g - \zeta^* &= \chi\eta\bar{c}_w(\Delta_\phi(\kappa) - 1) \\ &= -\frac{\chi\eta\varepsilon\bar{c}_w}{\kappa} \cdot \phi\Delta_\phi(\kappa) \\ &< 0 \quad \text{if } \phi > 0, \text{ and} \\ &= 0 \quad \text{if } \phi = 0.\end{aligned}$$

(iii). The difference of welfare between under no federal intervention and under the optimal uniform environmental standard is

$$W - W(\zeta^*) = \frac{((1-\sigma)\bar{W} + 1/\delta)}{1-\sigma} \cdot \Delta_\phi(1)[\Delta_\phi(\kappa)^{-\kappa} - \Delta_\phi(\sigma)^{-\sigma}].$$

Note that $\kappa - \sigma = -\chi\eta(1-\sigma)$. If $\sigma = 1$, then $W = W(\zeta^*)$ since $\kappa = \sigma$ and, thus, $\Delta_\phi(\kappa)^{-\kappa} = \Delta_\phi(\sigma)^{-\sigma}$ for any $\phi \geq 0$. Suppose that $\sigma < 1$ (or $\sigma > 1$). Then $W < W(\zeta^*)$ since $\Delta_\phi(\kappa)^{-\kappa} < \Delta_\phi(\sigma)^{-\sigma}$ (or $\Delta_\phi(\kappa)^{-\kappa} > \Delta_\phi(\sigma)^{-\sigma}$) with $\kappa < \sigma$ (or $\kappa > \sigma$) for any $\phi \geq 0$ from Definition 3.3.1. $\square$

Proof of Proposition 3.4.2. First, I prove that the parameter ι is positive. If $\sigma \geq 1$, then $\iota = 1 - \chi\eta(1-\sigma) > 0$. Suppose that $\sigma < 1$. Since $\kappa = 1 - (1+\chi\eta)(1-\sigma) > 0$ by Lemma 1 and $1-\sigma > 0$, we have that $0 < (1+\chi\eta)(1-\sigma) < 1$. Then $\iota = 1 - \chi\eta(1-\sigma) = 1 - (\chi\eta/(1+\chi\eta))(1+\chi\eta)(1-\sigma) > 0$. (i). The difference of growth rates between under no federal intervention and under the optimal requirement of lump sum transfer is

$$\begin{aligned}
\gamma - \gamma(\xi^*) &= -\frac{\bar{c}_w[(1+\chi\eta)\Delta_\phi(\kappa) - (1+\chi\eta\Delta_\phi(\iota))]}{\varepsilon} \\
&= \frac{\bar{c}_w}{\kappa\iota} \cdot \frac{\phi\Delta_\phi(\kappa)\Delta_\phi(\iota)}{\Delta_\phi(1)} \\
&> 0 \quad \text{if } \phi > 0, \text{ and} \\
&= 0 \quad \text{if } \phi = 0.
\end{aligned}$$

(ii). The difference of abatement-capital ratios between under no federal intervention and the optimal requirement of lump sum transfer is

$$\begin{aligned}
g - g(\xi^*) &= \chi\eta\bar{c}_w(\Delta_\phi(\kappa) - \Delta_\phi(\iota)) \\
&= -(1-\sigma) \cdot \frac{\chi\eta\bar{c}_w\varepsilon}{\kappa\iota} \cdot \phi\Delta_\phi(\kappa)\Delta_\phi(\iota) \\
&<, =, \text{or} > 0 \quad \text{for } \sigma <, =, \text{or} > 1 \quad \text{when } \phi > 0 \text{ and} \\
&= 0 \quad \text{when } \phi = 0.
\end{aligned}$$

(iii). The difference of welfare levels between under no federal intervention and under the optimal requirement of lump sum transfer is

$$W - W(\xi^*) = \frac{((1-\sigma)\bar{W} + 1/\delta)}{1-\sigma} \cdot \Delta(\phi, 1)[\Delta_\phi(\kappa)^{-\kappa} - \Delta_\phi(\iota)^{-\iota}].$$

If $\phi = 0$, then $W = W(\xi^*)$. Suppose that $\phi > 0$. Note that $\kappa - \iota = -(1-\sigma)$. If $\sigma = 1$, then $W = W(\xi^*)$ since $\kappa = \iota$ and, in turn, $\Delta_\phi(\kappa)^{-\kappa} = \Delta_\phi(\iota)^{-\iota}$

for any $\phi > 0$. Suppose that $\sigma < 1$ or ($\sigma > 1$). Then $W < W(\xi^*)$ since $\Delta_\phi(\kappa)^{-\kappa} < \Delta_\phi(\iota)^{-\iota}$ or ($\Delta_\phi(\kappa)^{-\kappa} > \Delta_\phi(\iota)^{-\iota}$) with $\kappa < \iota$ or ($\kappa > \iota$) for $\phi > 0$ from Definition 3.3.1. □

Proof of Corollary 3.4.3. The difference of welfare levels between under the optimal uniform standard of environmental quality and under the optimal requirement of lump sum transfer is

$$W(\zeta^*) - W(\xi^*) = \frac{((1-\sigma)\bar{W} + 1/\delta)}{1-\sigma} \cdot \Delta_\phi(1)[\Delta_\phi(\sigma)^{-\sigma} - \Delta_\phi(\iota)^{-\iota}].$$

Note that $\sigma - \iota = -(1-\chi\eta)(1-\sigma)$. If $\chi\eta = 1$ or $\sigma = 1$, then $W(\zeta^*) = W(\xi^*)$ since $\sigma = \iota$, and thus, $\Delta_\phi(\sigma)^{-\sigma} = \Delta_\phi(\iota)^{-\iota}$ for $\phi \in (0, \infty)$. Suppose that $\chi\eta < 1$ and $\sigma < 1$ (or $\sigma > 1$). Then $W(\zeta^*) < W(\xi^*)$ since $\sigma < \iota$ (or $\sigma > \iota$), and hence, $\Delta_\phi(\sigma)^{-\sigma} < \Delta_\phi(\iota)^{-\iota}$ (or $\Delta_\phi(\sigma)^{-\sigma} > \Delta_\phi(\iota)^{-\iota}$) for $\phi \in (0, \infty)$ by Definition 3.3.1. Suppose that $\chi\eta > 1$ and $\sigma < 1$ (or $\sigma > 1$). Then $W(\zeta^*) > W(\xi^*)$ since $\sigma > \iota$ (or $\sigma < \iota$), and thus, $\Delta_\phi(\sigma)^{-\sigma} > \Delta_\phi(\iota)^{-\iota}$ (or $\Delta_\phi(\sigma)^{-\sigma} < \Delta_\phi(\iota)^{-\iota}$) for $\phi \in (0, \infty)$ by Definition 3.3.1. Therefore, the comparison of two welfare is not independent of $\sigma \neq 1$. □

B.2 Derivations

B.2.1 Derivation of optimal uniform environmental standard

Local outcomes for a given uniform standard

Differentiating the current-value Hamiltonian (3.32) with respect to the capital tax rate τ, the sock of capital K, and the shadow price of capital μ, and then substituting the ex-post equilibrium conditions (3.13) and (3.14), we take the following first-order conditions:

$$U_C = \mu(1/\varepsilon + \phi)K, \tag{B.1}$$

$$((1-\alpha)A + \tau - \zeta)U_C - \mu(\alpha A - \tau - \rho)/\varepsilon = \delta\mu - \dot{\mu}, \tag{B.2}$$

$$\dot{K} = (1/\varepsilon)(\alpha A - \tau - \rho)K. \tag{B.3}$$

Differentiating eq. (B.1) with respect to time, and then replacing this result and eq. (B.1) into eq. (B.2), we get

$$\frac{\dot{C}_w}{C_w} = (((1-\alpha)A + \tau - \zeta)(1/\varepsilon + \phi) + (\alpha A - \tau - \rho)/\varepsilon - \delta)/\sigma, \tag{B.4}$$

which is the Keynes-Ramsey rule that describes the optimal saving-investment path for the capital stock under a uniform standard.

The optimal uniform standard

The total differentiation of the integrated life-time utility function (3.29) gives

$$\mathrm{d}W = (W(1-\sigma) + 1/\delta)\left(\frac{\mathrm{d}c_w}{c_w} - \eta\frac{\mathrm{d}P}{P} + \frac{\mathrm{d}\gamma}{\delta - \gamma(1-\sigma)}\right). \tag{B.5}$$

Differentiating totally the growth rate $\gamma(\zeta)$ in eq. (3.33), the consumption-capital ratio $c_w(\zeta)$ in eq. (3.35), and the pollution function (3.1), we have

$$\frac{\mathrm{d}\gamma}{\delta - \gamma(1-\sigma)} = -\frac{(1/\varepsilon)\mathrm{d}\zeta}{\delta - (1-\sigma)(A-\rho)/\varepsilon + \zeta(1-\sigma)/\varepsilon}. \tag{B.6}$$

$$\frac{\mathrm{d}c_w}{c_w} = \frac{(1/\varepsilon)(1-\sigma)\mathrm{d}\zeta}{\delta - (1-\sigma)(A-\rho)/\varepsilon + \zeta(1-\sigma)/\varepsilon}, \tag{B.7}$$

$$\frac{\mathrm{d}P}{P} = -\chi \frac{\mathrm{d}\zeta}{\zeta}. \tag{B.8}$$

By plugging eqs. (B.6), (B.7), and (B.8) into eq. (B.5), we arrive at the first-order condition (3.36) for the uniform standard ζ of environmental quality.

B.2.2 Derivation of optimal requirement of lump sum transfer (or tax)

Local outcomes for a given requirement

Differentiating the current-value Hamiltonian (3.44) with respect to the capital tax rate τ, the capital stock K, and the shadow price of capital μ, and then replacing the ex-post equilibrium conditions in eqs. (3.13) and (3.14), we get the first-order conditions as

$$U_P = -\mu(1/\varepsilon + \phi)K\chi(\tau - \xi)^{\chi+1}, \tag{B.9}$$

$$((1-\alpha)A + \xi)U_C + \mu(\alpha A - \tau - \rho)/\varepsilon = \delta\mu - \dot{\mu}, \tag{B.10}$$

$$\dot{K} = (1/\varepsilon)(\alpha A - \tau - \rho)K. \tag{B.11}$$

Differentiating eq. (B.9) with respect to time and plugging the derivative and eq. (B.9) into eq. (B.10) yield

$$\frac{\dot{C}_w}{C_w} = -\left(\frac{(1/\varepsilon + \phi)(\tau - \xi)}{\chi\eta} - \delta - (\eta(1-\sigma) + 1)\frac{\dot{P}}{P}\right)/(1-\sigma), \tag{B.12}$$

which is the Keynes-Ramsey rule that implies the optimal saving-investment path for the stock of capital under a federal requirement of transfer.

The optimal requirement

Differentiating totally the consumption-capital ratio $c_w(\xi)$ in eq. (3.43), the growth rate $\gamma(\xi)$ in eq. (3.45), the abatement-capital ratio $g(\xi)$ in eq. (3.47), and the pollution function (3.1), we have

$$\frac{\mathrm{d}c_w}{c_w} = \frac{\mathrm{d}\xi}{(1-\alpha)A+\xi}, \tag{B.13}$$

$$\frac{\mathrm{d}\gamma}{\delta-\gamma(1-\sigma)} = -\frac{\mathrm{d}g}{g(1-\sigma)}, \tag{B.14}$$

$$\frac{\mathrm{d}g}{g} = \frac{(1-\sigma)(1/\varepsilon)\mathrm{d}\xi}{\delta-(1-\sigma)(1/\varepsilon)(\alpha A-\rho-\xi)}, \tag{B.15}$$

$$\frac{\mathrm{d}P}{P} = -\chi\frac{\mathrm{d}g}{g}. \tag{B.16}$$

Hence, we arrive at eq. (3.48) by substituting eq. (B.15) into eqs. (B.14) and (B.16), and then plugging two results and eq. (B.13) into eq. (B.5).

Bibliography

Aghion, Philippe and Peter Howitt (1998), *Endogenous Growth Theory*, The MIT Press.

Allingham, Michael G. and Agnar Sandmo (1972), 'Income tax evasion: a theoretical analysis', *Journal of Public Economics* **1**(3-4), 323–338.

Alm, James (1985), 'The welfare cost of the underground economy', *Economic Inquiry* **23**(2), 243–263.

Aronsson, Thomas and Olof Johansson-Stenman (2008), 'When the joneses' consumption hurts: Optimal public good provision and nonlinear income taxation', *Journal of Public Economics* **92**(5-6), 986–997.

Arrow, Kenneth J. (1962), 'The economic implications of learning by doing', *Review of Economic Studies* **29**(3), 155–173.

Arrow, Kenneth J. (1971), *Essays in the Theory of Risk-Bearing*, Chicago: Markham Publishing Co.

Atkinson, A. B. and J. E. Stiglitz (1976), 'The design of tax structure: Direct versus indirect taxation', *Journal of Public Economics* **6**(1-2), 55–75.

Ballard, Charles L. and Don Fullerton (1992), 'Distortionary taxes and the provision of public goods', *Journal of Economic Perspectives* **6**(3), 117–131.

Barro, Robert J. and Xavier Sala i Martin (1992), 'Public finance in models of economic growth', *Review of Economic Studies* **59**, 645–661.

Barro, Robert J. and Xavier Sala i Martin (2004), *Economic Growth*, The MIT Press.

Bovenberg, A. Lans and Lawrence H. Goulder (1996), 'Optimal environmental taxation in the presence of other taxes: General-equilibrium analyses', *American Economic Review* **86**(4), 985–1000.

Bovenberg, A Lans and Ruud A de Mooij (1994), 'Environmental levies and distortionary taxation', *American Economic Review* **84**(4), 1085–89.

Bovenberg, A. Lans and Sjak Smulders (1995), 'Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model', *Journal of Public Economics* **57**(3), 369–391.

Browning, Edgar K. (1976), 'The marginal cost of public funds', *Journal of Political Economy* **84**(2), 283–98.

Browning, Edgar K. (1987), 'On the marginal welfare cost of taxation', *American Economic Review* **77**(1), 11–23.

Browning, Edgar K. (1994), 'The non-tax wedage', *Journal of Public Economics* **53**(3), 419–433.

Christiansen, Vidar (1980), 'Two comments on tax evasion', *Empirical Economics* **13**(3), 389–393.

Christiansen, Vidar (1984), ‘Which commodity taxes should supplement the income tax?’, *Journal of Public Economics* **24**(2), 195–220.

Christie, Edward and Mario Holzner (2006), What explains tax evasion? an empirical assessment based on european data, Working Paper 40, The Vienna Institute for International Economic Studies.

Cremer, Helmuth, Pierre Pestieau and Jean-Charles Rochet (2001), ‘Direct versus indirect taxation: The design of the tax structure revisted’, *International Economic Review* **42**(3), 781–99.

Cumberland, John H. (1979), *Interregional pollution spillovers and consistency of environmental policy*, New York University Press.

Cumberland, John H. (1981), ‘Efficiency and equity in interregional environmental management’, *Review of Regional Studies* (2), 1–9.

Dreze, Jean and Nicholas Stern (1990), ‘Policy reform, shadow prices, and market prices’, *Journal of Public Economics* **42**(1), 1–45.

Francesco (n.d.), ‘Channels of transmission of environmental policy to economic growth: a survey of the theory’, Mimeo.

Frank, Robert H. (1985), ‘The demand for unobservable and other nonpositional goods’, *American Economic Review* **75**(1), 101–16.

Frank, Robert H. (1999), *Luxury Fever*, The Free Press, New York.

Frankel, Jeffrey A. (2003), The environment and globalization, NBER Working Papers 10090, National Bureau of Economic Research, Inc.

Fullerton, Don (1997), 'Environmental levies and distortionary taxation: Comment', *American Economic Review* **87**(1), 245–51.

Fullerton, Don and Gilbert E. Metcalf (2001), 'Environmental controls, scarcity rents, and pre-existing distortions', *Journal of Public Economics* **80**(2), 249–267.

Goulder, Lawrence H. (1995), 'Environmental taxation and the double dividend: A reader's guide', *International Tax and Public Finance* **2**(2), 157–183.

Goulder, Lawrence H., Ian W.H. Parry and Dallas Burtraw (1997), 'Revenue-raising versus other approaches to environmental protection: The critical significance of preexisting tax distortions', *RAND Journal of Economics* **28**(4), 708–731.

Harberger, Arnold C. (1964), 'The measurement of waste', *American Economic Review Papers and Proceedings* **54**, 58–76.

Hausman, Jerry A. (1985), Taxes and labor supply, *in* A. J.Auerbach and M.Feldstein, eds, 'Handbook of Public Economics', Vol. 1 of *Handbook of Public Economics*, Elsevier, chapter 4, pp. 213–263.

Hayashi, Fumio (1982), 'Tobin's marginal q and average q: A neoclassical interpretation', *Econometrica* **50**(1), 213–224.

Hopkins, Ed and Tatiana Kornienko (2004), ‘Running to keep in the same place: Consumer choice as a game of status’, *American Economic Review* **94**(4), 1085–1107.

Ireland, N. J. (2001), ‘Optimal income tax in the presence of status effects’, *Journal of Public Economics* **81**(2), 193–212.

Javorcik, Beata and Shang-Jin Wei (2005), ‘Pollution havens and foreign direct investment: Dirty secret or popular myth?’, *Contributions to Economic Analysis & Policy* **3**(2), 1244–1244.

Judd, Kenneth L. (1985), ‘Redistributive taxation in a simple perfect foresight model’, *Journal of Public Economics* **28**, 29–53.

Kaplow, Louis (1996), ‘The optimal supply of public goods and the distortionary cost of taxation’, *National Tax Journal* **49**(4), 513–533.

Kaplow, Louis (2006), ‘On the undesirability of commodity taxation even when income taxation is not optimal’, *Journal of Public Economics* **90**(6-7), 1235–1250.

Kunce, Mitch and Jason F. Shogren (2005), ‘On interjurisdictional competition and environmental federalism’, *Journal of Environmental Economics and Management* (50), 212–224.

Lee, Dwight R. and Walter S. Misiolek (1986), ‘Substituting pollution taxation for general taxation: Some implications for efficiency in pollution taxation’, *Journal of Environmental Economics and Management* **13**(4), 338–347.

Lejour, Arjan M. and Harrie A. A. Verbon (1997), 'Tax competition and redistribution in a two-country endogenous-growth model', *International Tax and Public Finance* **4**, 485–497.

Ljungqvist, Lars and Harald Uhlig (2000), 'Tax policy and aggregate demand management under catching up with the joneses', *American Economic Review* **90**(3), 356–366.

Markusen, James R., Edward R. Morey and Nancy Olewiler (1995), 'Competition in regional environmental policies when plant locations are endogenous', *Journal of Public Economics* **56**(1), 55–77.

Mayshar, Yoram (1991), 'On measuring the marginal cost of funds analytically', *American Economic Review* **81**(5), 1329–1335.

Mirrlees, J. A. (1976), 'Optimal tax theory: A synthesis', *Journal of Public Economics* **6**(4), 327–358.

Oates, Wallace E. (1995), 'Green taxes: Can we protect the environment and improve the tax system at the same time?', *Southern Economic Journal* **61**(4), 915–922.

Oates, Wallace E. and Robert M. Schwab (1988), 'Economic competition among jurisdictions: Efficiency enhancing or distortion inducing?', *Journal of Public Economics* **35**(3), 333–354.

Parry, Ian W. H. (1995), 'Pollution taxes and revenue recycling', *Journal of Environmental Economics and Management* **29**(3), S64–S77.

Parry, Ian W.H. (1997), 'Environmental taxes and quotas in the presence of distorting taxes in factor markets', *Resource and Energy Economics* **19**(3), 203–220.

Parry, Ian W.H., Roberton C. Williams III and Lawrence H. Goulder (1999), 'When can carbon abatement policies increase welfare? the fundamental role of distorted factor markets', *Journal of Environmental Economics and Management* **37**(1), 52–84.

Pigou, Arthur C. (1947), *A Study in Public Finance*, third edition edn, London: Macmillan.

Pigou, Arthur Cecil (1920), *The Economics of Welfare*, Macmillan, London. (4th edition 1932).

Rauscher, Michael (2005), 'Economic growth and tax-competing leviathans', *International Tax and Public Finance* **12**(4), 457–474.

Razin, Assaf and Chi-Wa Yuen (1999), 'Optimal international taxation and growth rate convergence: Tax competition vs. coordination', *International Tax and Public Finance* **6**, 61–78.

Rebelo, Sergio (1991), 'Long-run policy analysis and long-run growth', *Journal of Political Economy* **99**, 500–521.

Repetto, Robert, Roger C. Dower, Robin Jenkins and Jacqueline Geoghegan (1992), *Green Fees: How a Tax Shift Can Work for the Environment and the Economy*, World Resources Institute, Washington, DC.

Robson, Arthur J. (1992), 'Status, the distribution of wealth, private and social attitudes to risk', *Econometrica* **60**(4), 837–57.

Saez, Emmanuel (2002), 'The desirability of commodity taxation under non-linear income taxation and heterogeneous tastes', *Journal of Public Economics* **83**(2), 217–230.

Sandmo, Agnar (1975), 'Optimal taxation in the presence of externalities', *Swedish Journal of Economics* **77**(1), 86–98.

Sandmo, Agnar (1981), 'Income tax evasion, labour supply, and the equity-efficiency tradeoff', *Journal of Public Economics* **16**, 265–288.

Slemrod, Joel (2001), 'A general model of the behavioral response to taxation', *International Tax and Public Finance* **8**(2), 119–128.

Slemrod, Joel and Shlomo Yitzhaki (1996), 'The costs of taxation and the marginal efficiency cost of funds', *IMF Staff Papers* **43**(1), 172–198.

Slemrod, Joel and Shlomo Yitzhaki (2002), Tax avoidance, evasion, and administration, *in* A. J.Auerbach and M.Feldstein, eds, 'Handbook of Public Economics', Vol. 3 of *Handbook of Public Economics*, Elsevier, chapter 22, pp. 1423–1470.

Smulders, Sjak and Raymond Gradus (1996), 'Pollution abatement and long-term growth', *European Journal of Political Economy* **12**, 505–532.

Stuart, Charles (1984), ‘Welfare costs per dollar of additional tax revenue in the united states’, *American Economic Review* **74**(3), 352–362.

Tahvonen, Olli and Jari Kuuluvainen (1991), ‘Optimal growth with renewable resources and pollution’, *European Economic Review* **35**(2-3), 650–661.

Tiebout, Charles M. (1956), ‘A pure theory of local expenditures’, *Journal of Political Economy* **64**(5), 416–424.

Veblen, Thorstein (1899), *The Theory of the Leisure class*, Macmillan, New York.

Wendner, Ronald and Lawrence H. Goulder (2008), Status effects, public goods provision, and the excess burden. MPRA Paper with number 8260, University Library of Munich, Germany.

Wildasin, David E. (1988), ‘Nash equilibria in models of fiscal competition’, *Journal of Public Economics* **35**, 229–240.

Wildasin, David E. (2003), ‘Fiscal competition in space and time’, *Journal of Public economics* **87**, 2571–2588.

Williams, Roberton C. III (2002), ‘Environmental tax interactions when pollution affects health or productivity’, *Journal of Environmental Economics and Management* **44**(2), 261–270.

Wilson, John D. (1986), ‘A theory of interregional tax competition’, *Journal of Urban Economics* **19**, 296–315.

Yitzhaki, Shlomo (1974), 'A note on income tax evasion: A theoretical analysis', *Journal of Public Economics* **3**(2), 201–202.

Yitzhaki, Shlomo (1987), 'On the excess burden of tax evasion', *Public Finance Review* **15**(2), 123–137.

Zodrow, George R. and Peter Mieszkowski (1986), 'Pigou, tiebout, property taxation, and the underprovision of local public goods: Comment', *Journal of Urban Economics* **19**, 356–370.

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